I'M THE BOSS OF MY BODY!

STOP SEXUAL ABUSE!

STOP SEXUAL ABUSE!
STOP TOUCHING ME!

TELL!
REPORT TO POLICE

IT IS EVERYONE'S RESPONSIBILITY TO STOP SEXUAL ABUSE

TALK TO PARENTS AND EDUCATORS

ABUSERS MUST BE REPORTED AND DEALT WITH LAWFULLY!

TELL! REPORT TO POLICE

STAY SAFE!

STOP SEXUAL ABUSE! STOP TOUCHING ME! RUN!

TELL! REPORT TO POLICE

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IT IS EVERYONE'S RESPONSIBILITY TO STOP SEXUAL ABUSE

TALK TO PARENTS AND EDUCATORS

ABUSERS MUST BE REPORTED AND DEALT WITH LAWFULLY!

TELL! REPORT TO POLICE

STAY SAFE!
These workbooks have been developed for the children of South Africa under the leadership of the Minister of Basic Education, Mrs Angie Motsepe, and the Deputy Minister of Basic Education, Dr Reginah Mhaule.

The Rainbow Workbooks form part of the Department of Basic Education’s range of interventions aimed at improving the performance of South African learners in the first six grades. As one of the priorities of the Government’s Plan of Action, this project has been made possible by the generous funding of the National Treasury. This has enabled the Department to make these workbooks, in all the official languages, available at no cost.

We hope that teachers will find these workbooks useful in their everyday teaching and in ensuring that their learners cover the curriculum. We have taken care to guide the teacher through each of the activities by the inclusion of icons that indicate what it is that the learner should do.

We sincerely hope that children will enjoy working through the book as they grow and learn, and that you, the teacher, will share their pleasure.

We wish you and your learners every success in using these workbooks.
Grade 7

Mathematics

Worksheets
65 to 144

Part 3

Name:
Describe each pattern.

Example:

We add six to 44 and get 50, we add another six to 50 and get 56. We subtract nine from 62 and get 53. We subtract another nine and get 44.

1. a. Describe the patterns involving adding and subtraction shown in the number line below.

2. b. Describe the patterns involving adding and subtraction shown in the number line below.

3. c. Describe the patterns involving adding and subtraction shown in the number line below.

4. d. Describe the patterns involving adding and subtraction shown in the number line below.
2. Describe the rule for each pattern.

Example: 27, 36, 45, 54, 63  
Rule: Adding 9 or counting on in 9s

a. 6, 14, 22, 30

b. 2, 6, 10, 14, 18

c. 13, 10, 7, 4, 1

d. 8, 13, 18, 23, 28

e. 5, 9, 13, 17, 21

f. –20, –15, –10, –5, 0

g. 7, 18, 29, 40, 51

h. 1, 9, 17, 25, 33

i. 4, 5, 6, 7, 8

j. –6, –4, –2, 0, 2

Sharing

The rule is ‘adding 11’. Start your pattern with 35.
Describe the pattern.
2, 4, 8, 16, ...

Identify the constant ratio between consecutive terms. This pattern can be described in your own words as “multiplying the previous number by 2”.

1. Describe the pattern.

Example: 8, 32, 128, 512
2 × 4 = 8
8 × 4 = 32
32 × 4 = 128
128 × 4 = 512

a. 2, 8, 32, 128, 512

b. 4, 12, 36, 108, 324

c. 6, 12, 24, 48, 96
Problem solving

If the rule is “subtracting 9”, give the first five terms of the sequence starting with 104.
Numeric patterns: neither a constant difference nor a constant ratio

What is the difference between constant difference and ratio?
- constant difference, e.g. 21, 23, 25, 27, ...
- constant ratio, e.g. 2, 4, 8, 16, ...

Describe the pattern.
1, 2, 4, 7, 11, 16, ...

This pattern has neither a constant difference nor a constant ratio. It can be described as “increasing the difference between consecutive terms by one each time” or “adding one more than was added to get the previous term”.

1. Describe the pattern and draw a number line to show each.

Example: 2, 4, 8, 14, 22

a. 8, 10, 14, 20, 28

b. 15, 12, 6, -3, -15

c. 3, 6, 10, 15, 21

d. 10, 9, 7, 4, 0

What will the next three terms be, applying the identified rule?

Take your time to figure out the pattern.
Problem solving

Create your own sequence without a constant ratio.

e. 6, 7, 9, 12, 16

f. 1, 3, 7, 15, 31

g. 13, 9, 4, –2, –9

h. 9, 14, 20, 27, 35

i. 24, 18, 13, 9, 6

j. 19, 20, 22, 25, 29
Numeric patterns: tables

Give a rule to describe the relationship between the numbers in this sequence: 2, 4, 6, 8, ... Use the rule to find the value of the tenth term.

The “tenth term” refers to position 10 in the number sequence. You have to find a rule in order to determine the value of the tenth term (rather than continuing the sequence up to the value of the tenth term). You should recognise that each term in the bottom row is obtained by doubling the number in the top row. So double 10 is 20. The tenth term is 20.

1. Describe the pattern and draw a number line to show each.

Example:

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>?</td>
</tr>
</tbody>
</table>

We can represent a sequence in a table.

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>1×3</td>
<td>2×3</td>
<td>3×3</td>
<td>4×3</td>
<td>10×3</td>
</tr>
</tbody>
</table>

a. | Position in the sequence | 1 | 2 | 3 | 4 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

b. | Position in the sequence | 1 | 2 | 3 | 4 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td></td>
</tr>
</tbody>
</table>

c. | Position in the sequence | 1 | 2 | 3 | 4 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td></td>
</tr>
</tbody>
</table>

d. | Position in the sequence | 1 | 2 | 3 | 4 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td></td>
</tr>
</tbody>
</table>

e. | Position in the sequence | 1 | 2 | 3 | 4 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
2. What will the term be?

Example: 5, 10, 15, 20. Position of the term × 5.

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>75</td>
</tr>
</tbody>
</table>

a. Position in the sequence | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>20</td>
</tr>
</tbody>
</table>

b. Position in the sequence | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>28</td>
</tr>
</tbody>
</table>

c. Position in the sequence | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>35</td>
</tr>
</tbody>
</table>

d. Position in the sequence | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>100</td>
</tr>
</tbody>
</table>

e. Position in the sequence | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>15</td>
<td>30</td>
<td>45</td>
<td>60</td>
<td>10</td>
</tr>
</tbody>
</table>

f. Position in the sequence | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td>50</td>
</tr>
</tbody>
</table>

Problem solving

Thabelo is building a model house from matches. If he uses 400 matches in the first section, 550 in the second and 700 in the third section, how many matches will he need to complete the fourth section, if the pattern continues?
Look at this pattern. What will the 20th term be?

4, 7, 10, 13, ...

If you consider only the relationship between consecutive terms, then you can continue the pattern ("adding 3 to previous number") up to the 20th term to find the answer. However, if you look for a relationship or rule between the term and the position of the term, you can predict the answer without continuing the pattern. Using number sequences can be useful for finding the rule.

First term: 4  = 3(1) + 1
Second term: 7  = 3(2) + 1
Third term: 10 = 3(3) + 1
Fourth term: 13 = 3(4) + 1

What will the 20th term be?

1. Look at the following sequences:
Describe the rule in your own words.
Calculate the 20th term using a number sequence

Example: Number sequence: 5, 7, 9, 11, ...
Rule in words: 2 × the position of the term + 3.
20th term: (2 × 20) + 3 = 43

a. Number sequence: 2, 5, 10, 17, ...
Rule:
20th term:

b. Number sequence: −8, −6, −4, −2, ...
Rule:
15th term:

c. Number sequence: −1, 2, 5, 8, ...
Rule:
12th term:

d. Number sequence: 6, 9, 12, 15, ...
Rule:
19th term:
e. **Number sequence**: –6, –2, 2, 6, …
   Rule:
   18th term:

f. **Number sequence**: 7, 12, 17, 22, …
   Rule:
   12th term:

h. **Number sequence**: –3, –1, 1, 3, …
   Rule:
   15th term:

i. **Number sequence**: 3, 7, 11, 15, …
   Rule:
   14th term:

j. **Number sequence**: 14, 24, 34, 44, …
   Rule:
   25th term:

**Problem solving**

Miriam collects stickers for her sticker album. If she collects 4 stickers on day 1, 8 on day 2, 16 on day 3 and 32 on day 4, how many will she collect on day 5 if the pattern continues?

Helen spends 2 hours playing computer games on the first day of the school holidays. On the second day she plays for 5 hours and on the third day she plays for 8 hours. For how many hours will she play on the fourth day if she kept on playing in this pattern?
What do you see? Describe the pattern.

Take your time to explore the pattern.

1. Create the first three terms of the following patterns with matchsticks and then draw the patterns in your book. Complete the tables.

a. Triangular pattern

<table>
<thead>
<tr>
<th>Position of a triangle in pattern</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Square pattern

<table>
<thead>
<tr>
<th>Position of a square in pattern</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
c. Rectangular pattern

<table>
<thead>
<tr>
<th>Position of a rectangle in pattern</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Pentagonal pattern

<table>
<thead>
<tr>
<th>Position of a pentagon in pattern</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Look at worksheets 65–70 again. Explain and give examples of the following:

**Arithmetic number patterns**
Deals with addition and subtraction. It is a sequence with a constant difference.

**Geometric number patterns**
Deals with multiplication and division. It is a sequence with a constant ratio.

**Problem solving**
Represent an octagonal number pattern.
Look at the example and describe it.

Adding 4 to the previous term
4 multiplied by the position of the term -1
4(n) – 1, where n is the position of the term.

Position in the sequence | 1 | 2 | 3 | 4
--- | --- | --- | --- | ---
Term | 3 | 7 | 11 | 15

First term: 3 = 4(1) – 1
Second term: 7 = 4(2) – 1
Third term: 11 = 4(3) – 1
Fourth term: 15 = 4(4) – 1

1. Describe the sequence in different ways using the template provided.

a. 5, 11, 17, 23, ...

i) 

ii)
Position in the sequence | 1 | 2 | 3 | 4
--- | --- | --- | --- | ---
Term | | | | |

iii) , where n is the position of the term.

First term: 
Second term: 
Third term: 
Fourth term: 
### b. 5, 7, 9, 11, ...

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**iii**), where \( n \) is the position of the term.

First term:        
Second term:       
Third term:        
Fourth term:       

### c. 10, 19, 28, 37, ...

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**iii**), where \( n \) is the position of the term.

First term:        
Second term:       
Third term:        
Fourth term:       

continued
**Numeric patterns: describe a pattern continued**

**d. 0, 4, 8, 12, ...**

i)  

ii)  

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

iii) , where $n$ is the position of the term.

First term:  
Second term: 
Third term:  
Fourth term: 

Find the pattern.
e. 14, 25, 36, 47, ...

i) 

ii) Position in the sequence | 1 | 2 | 3 | 4 
Term 

iii) , where \( n \) is the position of the term.

First term: 
Second term: 
Third term: 
Fourth term: 

Problem solving

What is the 30th term if the \( n^{th} \) position is \( 8(n) - 7 \)?
What do input and output mean? Make a drawing to show a real-life example.

Input → Process → Output

1. Complete the flow diagrams.
   a. 1 5 7 9 12 → × 6 → b
   b. 3 4 5 6 8 → × 4 → b
   c. 12 11 9 6 5 → × 8 → b
   d. 3 8 9 10 12 → × 9 → b

2. Use the given rule to calculate the value of b.
   Example:
   b = a × 4
   - 2 × 4 = 8
   - 3 × 4 = 12
   - 4 × 4 = 16
   - 5 × 4 = 20
   - 7 × 4 = 28

   a. b = a × 6
   b. b = a × 10
3. Use the given rule to calculate the unknown variable.

Example:

\[
\begin{array}{cc}
\text{a} & \text{b} \\
4 & 11 \\
6 & 15 \\
7 & 17 \\
8 & 19 \\
9 & 21 \\
\end{array}
\]

\[b = a \times 2 + 3\]

\[\begin{align*}
4 \times 2 + 3 &= 11 \\
6 \times 2 + 3 &= 15 \\
7 \times 2 + 3 &= 17 \\
8 \times 2 + 3 &= 19 \\
9 \times 2 + 3 &= 21 \\
\end{align*}\]

a. \[a = b \times 3 + 1\]

b. \[g = h \times 2 + 10\]

c. \[x = y \times 2 + 4\]

d. \[m = n + 7 \times 2\]

e. \[y = x + 0.5\]

f. \[b = a \times 0.2\]

g. \[r = t \times 1 + 5\]

h. \[v = w \times 3 + 8\]

Problem solving

Draw your own flow diagram where \(a = b + 7\).
Draw your own flow diagram where \(a = b \times 2 + 11\).
Discuss this:

The rule is $y = x + 5$

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>10</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>15</td>
<td>105</td>
</tr>
</tbody>
</table>

- $y = 1 + 5 = 6$
- $y = 2 + 5 = 7$
- $y = 3 + 5 = 8$
- $y = 10 + 5 = 15$
- $y = 100 + 5 = 105$

1. Complete the tables.

   a. $y = x + 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. $a = b + 7$

<table>
<thead>
<tr>
<th>$b$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. $m = n + 4$

<table>
<thead>
<tr>
<th>$n$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   d. $x = z \times 2$

<table>
<thead>
<tr>
<th>$z$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   e. $y = 2x - 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   f. $m = 3n + 2$

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>25</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What are the values of $m$ and $n$?

Example:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>18</th>
<th>$m$</th>
<th>51</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td></td>
<td>25</td>
<td>39</td>
</tr>
</tbody>
</table>

The rule: $y = x + 7$
Term 51: $y = 51 + 7$
   $y = 58$
   $\therefore n = 58$

The rule: $y = x + 7$
Term 39: $39 = x + 7$
   $39 - 7 = x + 7 - 7$
   $32 = x$
   $\therefore m = 32$

Rule: the given term plus 7

$n = 58$ and $m = 32$
**Problem solving**

- What is the tenth term in the pattern? \(3 \times 7, 4 \times 7, 5 \times 7, \ldots\)
- If \(x = 2y + 9\) and \(y = 2, 3, 4, 5, 6\) draw a table to show the values of \(x\) and \(y\).
What is on the left–hand side of the equal sign?

5 + 4

The left–hand side is an **expression**, 5 + 4. It is equal to the value of the expression 9.

What is on the right–hand side?

5 + 4 = 9

5 + 4 = 9 is called an **equation**. The left–hand side of an equation is equal to the right–hand side.

An equation is a mathematical sentence that uses the equal sign (=) to show that two expressions are equal.

1. Say whether it is an expression or an equation.

   **Example:**
   
   8 + 3 (It is an expression)
   8 + 3 = 11 (It is an equation)

   a. 4 + 8  
   b. 9 + 7 = 16  
   c. 7 + 6  

   d. 3 + 5 = 8  
   e. 11 + 2  
   f. 9 + 7  

2. Describe the following:

   **Example:**
   
   6 + 2 = 8  
   This is an **expression**, 6 + 2. It is equal to the value on the right–hand side, 8.  
   6 + 2 = 8 is called an **equation**. The left–hand side of an equation equals the right–hand side.

   a. 9 + 1 = 10  
   b. 3 + 5 = 8  
   c. 9 = 5 + 4  

23
Sign: ____________________
Date: ____________________
d. 7 = 1 + 6  e. 11 = 5 + 6  f. 8 + 9 = 17

3. Use the variable “a” to create 3 expressions of your own.
   Example: 5 + a

4. Say whether it is an expression or an equation.
   Example: 8 + a (It is an expression)
   8 + a = 11 (It is an equation)
   a. 5 + a  b. 6 + a = 12  c. 7 + b = 8
d. 8 + b  e. 9 + a = 18  f. 6 + b

5. What would the value of “a” be in question 4b, and e? ____________________

6. What would the value of “b” be in question 4c? ____________________

Problem solving
Write an equation for the following. I have 12 sweets. In total Phelo and I have 18 sweets. How many sweets does Phelo have?
1, 3, 5, 7, 9 …

Describe the rule of this number sequence in words.

What does the rule $2n - 1$ mean in the number sequence 1, 3, 5, 7, 9, …?

**Position in sequence** | 1   | 2   | 3   | 4   | 5   | $n$
---|---|---|---|---|---|---
**Value of term**   | 1   | 3   | 5   | 7   | 9   | $2n - 1$

**1. Describe the following in words:**

**Example:** 4, 8, 12, 16, 20, …

Adding 4 to the previous pattern

a. 3; 6; 9; 12; …
b. 10; 20; 30; 40; …
c. 7; 14; 21; 28; …
d. 6; 12; 18; 24; …
e. 8; 16; 24; 32; …
f. 5; 10; 15; 20; …

**2. Describe the following sequence using an expression.**

**Example:** 4, 8, 12, 16, 20, …

First term: $4(1) + 1$

The $n^{th}$ term is $4n$.

| Position in sequence | 1   | 2   | 3   | 4   | 5   | $n$
---|---|---|---|---|---|---
| Value of term        | 4   | 8   | 12  | 16  | 20  | $4n$ |

a. 6; 11; 16; 21; …
b. 3; 5; 7; 9; 11; ...

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. 9; 15; 21; 27; ...

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. What does the rule mean? Use the same values for position as in the example.

Example: The rule $2n - 1$ means for the following number sequence: 1, 3, 5, 7, 9 ...

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

a. The rule $3n - 1$ means for the following number sequence

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. The rule $4n - 3$ means for the following number sequence

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. The rule $6n - 2$ means for the following number sequence

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. The rule $5n - 5$ means for the following number sequence

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

e. The rule $7n - 4$ means for the following number sequence

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem solving
Write an algebraic expression for the following: Sipho built 3 times more puzzles than I did last holiday.
Describe the rule of this number sequence in words.

What does the rule $4n + 1$ mean for the number sequence 5, 9, 13, 17, 21, ...?

First term: $4(1) + 1$
Second term: $4(2) + 1$
Third term: $4(3) + 1$
Fourth term: $4(4) + 1$
Fifth term: $4(5) + 1$
$n^{th}$ term: $4(n) + 1$

Adding 2 to the previous term.

The rule as an expression

1. Describe the following in words:

   Example: 2, 6, 10, 14, 18, ...
   Adding 4 to the previous number

   a. 3; 5; 7; 9; ...
   b. 5; 10; 15; 20; ...
   c. 21; 18; 15; 12; ...
   d. 99; 98; 97; 96; ...
   e. 4; 8; 12; 16; ...
   f. 7; 14; 21; 28; ...

2. Describe the following sequence using an expression:

   Example: 2, 6, 10, 14, 18, ...
   $4n - 2$ since 1st term: $4(1) - 2$; 2nd term: $4(2) - 2$; Third term $4(3) - 2$; ...

   a. 2; 4; 6; 8; 10; ...
   b. 3; 5; 7; 9; 11; ...
   c. 8; 16; 24; 32; ...
   d. 5; 10; 15; 20; ...
Problem solving

If the rule is "adding $\frac{1}{4}$", what could the sequence be? Create five possible answers.

3. If the rule is ____, what could the sequence be? Create five possible answers for each.

a. “Adding 7”

b. “Subtracting 9”

c. “Adding 5”

d. “Subtracting 8”

e. “Adding 3 then subtracting 4”
Look at and describe:

- variable
- constants
- operation
- equal sign

\[ x + 23 = 45 \]

Read and answer:

Imagine that on the right-hand side of this balance scale there are 10 objects of equal mass, and on the left-hand side there are 4 similar objects and an unknown number of other objects in a bag. The scale is balanced; therefore, we know that there must be an equal mass on each side of the scale. Explain how you would find out how many objects there are in the bag.

1. Solve for \( x \).

Example:

\[ x + 5 = 9 \]

\[ x + 5 - 5 = 9 - 5 \]

\[ x = 4 \]

a. \( x + 12 = 30 \)

b. \( x + 8 = 14 \)

c. \( x + 17 = 38 \)

d. \( x + 20 = 55 \)

e. \( x + 25 = 30 \)

f. \( x + 18 = 26 \)

2. Solve for \( x \).

Example:

\[ x - 5 = 2 \]

\[ x - 5 + 5 = 2 + 5 \]

\[ x = 7 \]

a. \( x - 7 = 5 \)

b. \( x - 3 = 1 \)
29

Sign: Date:

Problem solving

Jason read 7 books and Gugu read 11 books. How many books did they read altogether?

Rebecca and her friend read 29 books altogether. Rebecca read 14 books. How many books did her friend read?

Bongani buys 12 new CDs and Sizwe buys 14. How many CDs did they buy together?

3. Solve for x.

Example: \( x + 4 = -7 \)
\[ x + 4 - 4 = -7 - 4 \]
\[ x = -11 \]

a. \( x + 3 = -15 \)

b. \( x + 7 = -12 \)

c. \( x + 2 = -5 \)

d. \( x + 5 = -15 \)

e. \( x + 12 = -20 \)

f. \( x + 10 = -25 \)

Problem solving

Write an equation for the following and solve it.

Jason read 7 books and Gugu read 11 books. How many books did they read altogether?

Rebecca and her friend read 29 books altogether. Rebecca read 14 books. How many books did her friend read?

Bongani buys 12 new CDs and Sizwe buys 14. How many CDs did they buy together?
More algebraic equations

2x = 30
What does 2x mean?
What is the inverse operation of multiplication?
We need to divide 2x by 2 to solve for x.

\[
\frac{2x}{2} = \frac{30}{2}
\]

\[x = 15\]

(2x means 2 multiplied by x)

Division

Remember you need to keep the two sides of the equation balanced. What you do on the one side of the equal sign, you must do on the other side as well.

1. Solve for x.

Example:

\[
3x = 12
\]

\[
\frac{3x}{3} = \frac{12}{3}
\]

\[x = 4\]

- a. 5x = 20
- c. 2x = 18
- e. 3x = 27
- g. 10x = 100
- i. 15x = 45

- b. 2x = 8
- d. 4x = 48
- f. 5x = 30
- h. 9x = 81
- j. 7x = 14
2. Solve for x.

**Example:**

\[ 3x - 2 = 10 \]
\[ 3x - 2 + 2 = 10 + 2 \]
\[ \frac{3x}{3} = \frac{12}{3} \]
\[ x = 4 \]

a. \[ 7x - 2 = 12 \]

b. \[ 4x - 4 = 12 \]

c. \[ 3x - 1 = 2 \]

d. \[ 2x - 1 = 7 \]

e. \[ 5x - 3 = 17 \]

f. \[ 5x - 7 = 13 \]

g. \[ 6x - 5 = 25 \]

h. \[ 9x - 8 = 82 \]

i. \[ 8x - 7 = 49 \]

j. \[ 3x - 2 = 16 \]

---

**Problem solving**

Create an equation and solve it. How fast can you do it?

- Two times y equals sixteen.
- Sixteen times b equals four.
- Nine times q equals eighty-one.
- Five times c equals sixty-five.
- Eight times t equals eighty.
- Five times y equals one-hundred.
- Eight times x equals sixteen.
- Three times d equals thirty-nine.
- Seven times a equals twenty-one.
Algebraic equations in context

What do the following equations mean?

\[ P = 4l \]
\[ P = 2l + 2b \]
\[ A = l^2 \]
\[ A = l \times b \]

The perimeter of a square is 4 times the length.

The perimeter of a rectangle is 2 times the length plus 2 times the breadth.

The area of a square is the length squared.

The area of a rectangle is length times breadth.

Note that you did perimeter and area in the first and second terms of grade 7.

1. Substitute and calculate.

Example: If \( y = x^2 + 2 \), calculate \( y \) when \( x = 4 \)

\[ y = 4^2 + 2 \]
\[ y = 16 + 2 \]
\[ y = 18 \]

a. \( y = x^2 + 2; \ x = 4 \)

b. \( y = b^2 + 10; \ b = 1 \)

c. \( y = a^2 + 4; \ a = 4 \)

d. \( y = r^2 + 3; \ r = 5 \)

e. \( y = p^2 + 7; \ p = 6 \)

f. \( y = c^2 + 7; \ c = 7 \)

2. Calculate the following:

Example: What is the perimeter of a rectangle if the length is 2 cm and the breadth is 1.5 cm?

\[ P = 2l + 2b \]
\[ P = 2(2 \text{ cm}) + 2(1.5 \text{ cm}) \]
\[ P = 4 \text{ cm} + 3 \text{ cm} \]
\[ P = 7 \text{ cm} \]
<table>
<thead>
<tr>
<th>Problem Solving</th>
</tr>
</thead>
</table>

Write an equation and then solve it for each of these:

- What is the perimeter of a rectangular swimming pool if the breadth is 12 m and the length is 16 m?
- Work out the area of a square if one side is equal to 5.2 cm.
- What is the perimeter of a rectangle if the length is 5.1 cm and the breadth is 4.9 cm.
- Establish the area of your rectangular bedroom floor for new tiles if the length is 4.5 m and the breadth is 2.8 m.

- a. The perimeter of a rectangle where the breadth equals 2.2 cm and the length equals 2.5 cm.
- b. The area of a square if the length equals 3.5 cm.
- c. The perimeter of a square if the length equals 4.2 cm.
- d. The area of a rectangle if the length is 3.5 cm and breadth is 2.5 cm.
- e. The area of a square if the length is 5 cm.
- f. The perimeter of a rectangle if the breadth is 4.3 cm and length 8.2 cm.
- g. The perimeter of a square if the length is 2.6 cm.
- h. The perimeter of a rectangle if the breadth is 8.5 cm and the length is 12.4 cm.
- i. The area of a rectangle if the breadth is 10.5 cm and length is 15.5 cm.
- j. The perimeter of a rectangle if the breadth is 3.5 cm and the length is 6.7 cm.
Look at the graph and discuss it.

Would you make any changes or add anything to the graph?

1. Thebogo heard that nature lovers use the chirping of crickets to estimate the temperature. The last time he went camping he brought a thermometer so he could collect the data on the number of cricket chirps per minute for various temperatures. The first thing Thebogo did was make the graph below.

Thebogo heard that nature lovers use the chirping of crickets to estimate the temperature. The last time he went camping he brought a thermometer so he could collect the data on the number of cricket chirps per minute for various temperatures. The first thing Thebogo did was make the graph below.
a. Estimate the temperature if the cricket chirps:
   i. 120 times? ____
   ii. 150 times? ____
   iii. 160 times? ____
   iv. 230 times? ____
   v. 270 times? ____

b. Thebogo counts 190 cricket chirps in a minute. What is the temperature?


c. Thebogo notices that the number of cricket chirps drops by 30 chirps per minute. What could she conclude about the change in temperature?


d. Use the words increasing and decreasing to describe the graph.
2. Average temperature per annum for Johannesburg, Cape Town and Durban:

a. What is the average maximum temperature for:
   i. Durban in August? ________
   ii. Cape Town in July? ________
   iii. Johannesburg in April? ________
   iv. Durban in July? ________
   v. Cape Town in September? ________

b. What is the average minimum temperature for:
   i. Johannesburg in April? ________
   ii. Cape Town in October? ________
   iii. Johannesburg in September? ________
   iv. Durban in March? ________
   v. Cape Town in July? ________
c. What is the difference in maximum temperature between:
   i. Durban and Johannesburg in April? ________
   ii. Cape Town and Durban in October? ________
   iii. Johannesburg and Cape Town in May?_______
   iv. Durban and Johannesburg in September? ________
   v. Cape Town and Johannesburg in April? ________

d. Describe the graphs using the words “increasing” and “decreasing”:

Problem solving

What is the difference between the minimum and maximum temperatures of Durban, Cape Town and Johannesburg in December? Which province would you most like to visit in December. Why?
Interpreting graphs: rainfall and time graphs

How do you read information from and interpret the graphs on this page.

1. Look at the graphs and answer the following questions:

   a. What is the heading of each graph?

   b. What does the x-axis show us?

   c. What does the y-axis show us?

   d. Which city has the highest average rainfall in October?
2. Use the graphs to complete the following tables:

<table>
<thead>
<tr>
<th>Months</th>
<th>Average rainfall</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Johannesburg</td>
<td>Durban</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>

e. Which city has the lowest average rainfall in April?

f. Which city would you visit in December? Why?

g. Which city would you not visit in December? Why?

h. Which city or cities have a rainy winter season? Why do you say so?

i. Which city or cities have a rainy summer season? Why do you say so?

j. Use the words increasing and decreasing to describe each graph.

Investigate the rainfall in your area.

What is the highest rainfall per year for your town? Which month? Keep a record during a rainy month and draw a graph to represent the data.
1. Use the graph below to answer the following questions on the movement of a snail.

\[ \text{Movement of a snail} \]

\[ \begin{array}{c}
\text{Time (hours)} \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\end{array} \]

\[ \begin{array}{c}
\text{Distance (cm)} \\
0 & 20 & 40 & 60 & 80 & 100 & 120 & 140 & 160 & 180 & 200 \\
\end{array} \]

a. How far will a snail move in eight hours?

b. How far will a snail move in four hours? How did you use the graph to work this out?

c. How far will a snail move in six hours? How did you use the graph to work this out?

d. How far will a snail move in two hours? How did you use the graph to work this out?

e. How far will a snail move in 9 hours? How did you use the graph to work this out? Plot this on the graph.

f. Why is this a linear graph?

g. Is this graph increasing or decreasing?
2. The graph below shows the distances travelled by car from Bloemfontein to Cape Town.

![Graph of Travel Distances from Bloemfontein to Cape Town]

How long did it take the person to travel ____ km? Show the position on the graph and explain it.

**Example:**

- **1 000 km**
  - It took the person ten hours to travel 1 000 km.
  - We can write it as (1 000 km/10 hours).

3. How far did the person travel in:

   a. 100 km
   b. 500 km
   c. 800 km
   d. 750 km
   e. 300 km
   f. 250 km

3. How far did the person travel in:

   a. 1 hour
   b. 1 hour 30 minutes
   c. 3 hours
   d. 4 hours 30 minutes
   e. 5 hours
   f. 2 hours 30 minutes

**How long did you travel?**

Use the graph on “Travelling from Bloemfontein to Cape Town” to work out how long it will take to travel 275 km.
1. Answer the questions on the graph below.

You kept this record but forgot to plot the minimum temperature. Plot it using the information from your notes.

**Average maximum temperature for our town**

<table>
<thead>
<tr>
<th>Months</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>30</td>
</tr>
<tr>
<td>F</td>
<td>25</td>
</tr>
<tr>
<td>M</td>
<td>20</td>
</tr>
<tr>
<td>A</td>
<td>15</td>
</tr>
<tr>
<td>M</td>
<td>10</td>
</tr>
<tr>
<td>J</td>
<td>5</td>
</tr>
<tr>
<td>A</td>
<td>10</td>
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<td>S</td>
<td>15</td>
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<td>O</td>
<td>20</td>
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<tr>
<td>N</td>
<td>25</td>
</tr>
<tr>
<td>D</td>
<td>30</td>
</tr>
</tbody>
</table>

**Average minimum temperature for our town**

<table>
<thead>
<tr>
<th>Months</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January:</td>
<td>20°C</td>
</tr>
<tr>
<td>February:</td>
<td>19°C</td>
</tr>
<tr>
<td>March:</td>
<td>15°C</td>
</tr>
<tr>
<td>April:</td>
<td>12°C</td>
</tr>
<tr>
<td>May:</td>
<td>10°C</td>
</tr>
<tr>
<td>June:</td>
<td>5°C</td>
</tr>
<tr>
<td>July:</td>
<td>4°C</td>
</tr>
<tr>
<td>August:</td>
<td>6°C</td>
</tr>
<tr>
<td>September:</td>
<td>9°C</td>
</tr>
<tr>
<td>October:</td>
<td>12°C</td>
</tr>
<tr>
<td>November:</td>
<td>15°C</td>
</tr>
<tr>
<td>December:</td>
<td>18°C</td>
</tr>
</tbody>
</table>

**Graph Questions**

- What is the average maximum temperature for our town in January?
- What is the average minimum temperature for our town in July?
- In which month did the temperature drop to 5°C?
a. What is the heading of the graph?

b. What is the scale on the x-axis?

c. What is the scale on the y-axis?

d. What does the x-axis tell us?

e. What does the y-axis tell us?

f. What do the points or dots tell us?
2. Use the grid paper on the next page to draw a graph for this table.

<table>
<thead>
<tr>
<th>Month</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>30</td>
<td>16</td>
</tr>
<tr>
<td>F</td>
<td>29</td>
<td>17</td>
</tr>
<tr>
<td>M</td>
<td>28</td>
<td>14</td>
</tr>
<tr>
<td>A</td>
<td>26</td>
<td>12</td>
</tr>
<tr>
<td>M</td>
<td>24</td>
<td>8</td>
</tr>
<tr>
<td>J</td>
<td>21</td>
<td>6</td>
</tr>
<tr>
<td>J</td>
<td>21</td>
<td>5</td>
</tr>
<tr>
<td>A</td>
<td>22</td>
<td>6</td>
</tr>
<tr>
<td>S</td>
<td>24</td>
<td>8</td>
</tr>
<tr>
<td>O</td>
<td>25</td>
<td>12</td>
</tr>
<tr>
<td>N</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>D</td>
<td>28</td>
<td>15</td>
</tr>
</tbody>
</table>

Use the whole sheet to draw your graph.

You should determine your intervals carefully.

a. What will you write on your x-axis?

b. What will you write on your y-axis?

c. What will the scale of the y-axis be?

d. What will the heading of your graph be?

e. What will your graph show?

f. Describe the graph using the following words: increasing or decreasing, linear or non-linear.
f. Describe the graph using the following words: increasing or decreasing, linear or non-linear.

Problem solving

Expand the following and prove your answer by factorising. \[ 2(p^3 + 8p^2 - 5p) \]

Research

Draw a graph showing the monthly maximum and minimum temperatures for any country other than South Africa, for one year.
You have to draw a graph with the following values. How will you do it?
The maximum value of the y–axis is 24.
The maximum value of the x–axis is 60.

The scale could be:
5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60

The scale could be:
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

Why are these intervals in 5s and not in 2s or 10s?

1. In this activity you should use the grid paper to draw the scales of your graph. Determine the scale for the y–axis and the x–axis.

The maximum value of:

a. x–axis is 45 and y–axis is 24

b. x–axis is 75 and y–axis is 72

c. x–axis is 40 and y–axis is 30

d. x–axis is 100 and y–axis is 100
2. Draw the scales for the following graphs:

   a. x–axis: 0, 3, 6, 9, 12, 15 and y–axis: 0, 5, 10, 15, 20, 25, 30

   b. x–axis: 0, 4, 8, 12 and y–axis: 0, 10, 20, 30, 40, 50, 60

   c. x–axis: 0, 5, 10, 15, 20, 25, 30, 35, 40 and y–axis: 0, 20, 40, 60, 80, 100

   d. x–axis: 36, 48, 60, 72, 84 and y–axis: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

3. Cut and paste a graph from a newspaper. Describe the intervals.

Drawing graphs

Draw a graph with 10 intervals on the x–axis and 12 intervals on the y–axis. You can use any multiples to label it.
1. Draw graphs using the data from the following tables. Describe each graph using the words increasing, decreasing, constant, linear and non-linear.

a. Thabo’s brisk walking results

The time walked was recorded after 2, 4, 6, 8 and 10 km.

<table>
<thead>
<tr>
<th>Km</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

Look at the graphs. Explain them.

Can you get a non-linear increasing graph?

Can you get a non-linear decreasing graph?
b. Susan’s brisk walking results

The time walked was recorded after 2, 4, 6, 8 and 10 km.

<table>
<thead>
<tr>
<th>Km</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
</tr>
<tr>
<td>10</td>
<td>95</td>
</tr>
</tbody>
</table>

c. Maximum and minimum average temperatures for a town for a year.

<table>
<thead>
<tr>
<th>Month</th>
<th>Minimum in degrees Celsius</th>
<th>Maximum in degrees Celsius</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>27</td>
<td>14</td>
</tr>
<tr>
<td>February</td>
<td>25</td>
<td>14</td>
</tr>
<tr>
<td>March</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>April</td>
<td>22</td>
<td>10</td>
</tr>
<tr>
<td>May</td>
<td>19</td>
<td>9</td>
</tr>
<tr>
<td>June</td>
<td>17</td>
<td>8</td>
</tr>
<tr>
<td>July</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td>August</td>
<td>17</td>
<td>8</td>
</tr>
<tr>
<td>September</td>
<td>22</td>
<td>9</td>
</tr>
<tr>
<td>October</td>
<td>23</td>
<td>12</td>
</tr>
<tr>
<td>November</td>
<td>25</td>
<td>13</td>
</tr>
<tr>
<td>December</td>
<td>28</td>
<td>14</td>
</tr>
</tbody>
</table>

Be creative

Create your own table, draw a graph and describe it.
## Transformations

### Explain each transformation

<table>
<thead>
<tr>
<th>Translation</th>
<th>Reflection</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram]</td>
<td>[Diagram]</td>
<td>[Diagram]</td>
</tr>
</tbody>
</table>

### 1. Say how each figure was moved. Write translation, rotation, or reflection.

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
<th>c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram]</td>
<td>[Diagram]</td>
<td>[Diagram]</td>
</tr>
</tbody>
</table>

### 2. Label each diagram as a translation, a reflection or a rotation of the original shape.

**Example:**

- Original shape
- Rotation
- Translation
- Reflection

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram]</td>
<td>[Diagram]</td>
</tr>
</tbody>
</table>

**Original shape**

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram]</td>
<td>[Diagram]</td>
</tr>
</tbody>
</table>

**Original shape**

Can you still remember these?
3. Create diagrams to the show:

a. Rotation

A rotation is a transformation that moves points so that they stay the same distance from a fixed point (the centre of rotation).

b. Reflection

A reflection is a transformation that has the same effect as a mirror image.

c. Translation

A translation is the movement of an object to a new position without changing its shape, size or orientation. When a shape is transformed by sliding it to a new position, without turning, we say it has been translated.

Problem solving

Create a diagram using reflection, rotation and translation.
**Rotation**

A rotation is a transformation that moves points so that they stay the same distance from a fixed point, the centre of rotation.

**Rotational symmetry:** A figure has rotational symmetry if an outline of the turning figure matches its original shape.

**Order of symmetry:** This is how many times an outline matches the original in one full rotation.

Use any **recycled material** to demonstrate the difference between rotation and rotational symmetry.

1. **Look at the diagrams and explain them in your own words**

   **Example:**
   
   \[
   \frac{1}{4} \text{ turn} = 90^\circ
   \]

   The paper rotated a quarter turn, which is the same as 90º. We can show this on a circular protractor.

   a. \(\frac{1}{2} \text{ turn} = 180^\circ\)
2. Look at the drawings below and explain them.

a. 

b. \( \frac{3}{4} \) turn = 270°

(c. 1 full turn = 360°

3. Complete the table below by rotating each shape and draw the rotated shape.

<table>
<thead>
<tr>
<th>Shape</th>
<th>90°</th>
<th>180°</th>
<th>270°</th>
<th>360°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem solving

Make up your own rotations, with the centre of rotation outside the shape.
1. Explain each translation in your own words. The original shape is shaded.

Example:

A translation is the movement of an object to a new position without changing its shape, size or orientation.

When a shape is transformed by sliding it to a new position, without turning, it is said to have been translated.

Each point of the triangle is translated four squares to the right and five squares up.

a.

b.
2. Show the following translations on a grid board.
   a. Each point of the triangle is translated four squares to the right and five squares up.
   b. Each point of the rectangle is translated three squares to the left and three squares up.
   c. Each point of the triangle is translated five squares to the right and two squares down.
   d. Each point of the square is translated two squares to the right and seven squares up.

3. In mathematics, the translation of an object is called its image. Describe the translation below.

   a. Each point of the triangle is translated four squares to the right and five squares up.
   b. Each point of the rectangle is translated three squares to the left and three squares up.
   c. Each point of the triangle is translated five squares to the right and two squares down.
   d. Each point of the square is translated two squares to the right and seven squares up.

Problem solving

Find a translated pattern in nature and explain it in words.
Reflection: a reflection is a transformation that has the same effect as a mirror image.

Reflective symmetry
An object is symmetrical when one half is a mirror image of the other half.

1. How many lines of symmetry does each of these have? Draw them in.

2. Draw all the lines of symmetry for each figure, where applicable.

   a.  
   b.  
   c.  

   d.  
   e.  
   f.  

3. The following design uses reflective symmetry. One half is a reflection of the other half. The two halves are exactly alike and fit perfectly on top of each other when the design is folded correctly. How many lines of symmetry are there?
4. Show a reflection using the geometric figure given. Remember to show the line of reflection.

a. 

b. 

c. 

d. 

e. 

f. 

5. Look at the reflections and describe them.

Problem solving

Find a photograph of reflection in nature.
Transformations again

Copy each transformation on grid paper and then explain it in words.

Rotation

Turning around a centre. The distance from the centre to any point on the shape stays the same. Every point makes a circle around the centre (rotation).

Reflection

It is a flip over a line. Every point is the same distance from the centre line. It has the same size as the original image. The shape stays the same (reflection).

Translation

It means moving without rotating, flipping or resizing. Every point of the shape must move the same distance and in the same direction (translation).

1. Describe each diagram. Make use of words such as mirror, shape, original shape, line of reflection and vertical.

Reflection

a. When a shape is reflected across a mirror line, the reflection is the same distance from the line of reflection as the original shape.

b.

c.
Rotation
Make use of words such as rotated or turned, clockwise, anti-clockwise, point of rotation and distance.

d.

f.

e.

Translation
Make use of words such as shape, slide, from one place to another, no turning, left, right, up or down.

g.

h.

g.

h.

Share with your family

Draw any shape and then do the following and describe the transformation:

• reflection • rotation • translation
When we do an investigation we should:
- spend enough time exploring problems in depth
- find more than one solution to many problems
- develop your own strategies and approaches, based on your knowledge and understanding of mathematical relationships
- choose from a variety of concrete materials and appropriate resources
- express your mathematical thinking through drawing, writing and talking.

1. Prove that the diagonal of a square is not equal to the length of any of its sides.

a. Make a drawing to show each of the following:

<table>
<thead>
<tr>
<th>What transformation is (rotation, reflection, and translation)</th>
<th>What a square is</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image" alt="Square" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>What diagonal lines of a square are</th>
<th>That all the sides of a square are equal in length</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagonal line" /></td>
<td></td>
</tr>
</tbody>
</table>

b. What do I want?
To compare the length of a side of a square with the length of a diagonal. I can/must use rotation, translation and/or reflection.

c. What do I need to introduce? Make a drawing of each.

Note that sometimes we think of something later on; we don’t always think of everything at the beginning. Therefore people will have different answers here.

<table>
<thead>
<tr>
<th>A line of reflection.</th>
<th>A point of rotation.</th>
<th>A grid on which to measure translation.</th>
</tr>
</thead>
</table>
d. Attack
We often get “stuck” and are tempted to give up. However, this is the exact point at which it is important for you to use the time and space to get through the point of frustration and look for alternative ideas. This is the phase when we make conjectures, collect data, discover patterns and try to justify our answers.

Remember to use the information in a, b and c.

e. Review
Check your conclusions or solutions and reflect on what you did – the key ideas and key moments.

Family time
Share this investigation with a family member.
Look at this diagram and discuss it.

Orange rectangle
The length = 5  The width = 3

Blue rectangle
The length = 10  The width = 6

The length of the blue rectangle is two times/twice the length of the orange rectangle.
The width of the blue rectangle is two times/twice the width of the orange rectangle.
The orange rectangle is enlarged twice/two times.

1. Use the diagrams to answer the questions.

a. Blue square
   Length = __
   Width = __

b. The length of the red square is ____ times the length of the blue square.
The width of the red square is ____ times the width of the blue square.
The red square is enlarged ____ times.

c. The length of the green square is ___ times the length of the red square rectangle.
The width of the green square is ___ times the width of the red square.
The green square is enlarged ____ times.

d. The length of the green square is ___ times the length of the blue square.
The width of the green square is ___ times the width of the blue square.
The blue square is reduced ___ times.
2. Use the diagrams to answer the questions.

Blue rectangle:
3 cm
The length = ___
The width = ___

Red rectangle:
6 cm
The length = ___
The width = ___

Green rectangle:
24 cm
8 cm
The length = ___
The width = ___

Compared to the:

a. Red rectangle, the blue rectangle is reduced ___ times.
b. Green rectangle, the blue rectangle is reduced ___ times.
c. Blue rectangle, the red rectangle is enlarged ___ times.
d. Green rectangle, the red rectangle is reduced ___ times.
e. Blue rectangle, the green rectangle is enlarged ___ times.
f. Red rectangle, the green rectangle is enlarged ___ times.

3. Draw a 1 cm by 2 cm rectangle. Enlarge it twice and then enlarge the second rectangle six times. Make a drawing to show your answer.

Problem solving

What will the perimeter of a 20 mm by 40 mm rectangle be if you enlarge it by 3?
More enlargement and reduction

How do you know this figure is enlarged by 3?

The scale factor from small to large is 3.
The scale factor from large to small is 3.

We say the scale factor is 3.

1. By what is this shape enlarged? Write down all the steps.

2. Enlarge the rectangle by:
   a. scale factor 4
   b. scale factor 3
3. Complete the table. Start with the original geometric figure each time. Your drawings do not have to be to scale but must be labelled correctly.

<table>
<thead>
<tr>
<th>Geometric figure</th>
<th>Enlarge by scale factor 2.</th>
<th>Enlarge by scale factor 5.</th>
<th>Enlarge by scale factor 10.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 2 cm x 3 cm</td>
<td>2 cm x 2 x 3 cm x 2</td>
<td>2 cm x 5 x 3 cm x 5</td>
<td>2 cm x 10 x 3 cm x 10</td>
</tr>
<tr>
<td></td>
<td>= 6 cm²</td>
<td>= 150 cm²</td>
<td>= 600 cm²</td>
</tr>
<tr>
<td>b. 5 cm x 1 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. 4 cm x 2 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. 8 cm x 3 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. 1.5 cm x 2 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem solving

Enlarge a 1.5 cm by 5 cm geometric figure by scale factor 3.
Enlargements and reductions

Use the knowledge you gained in the previous two worksheets. You might need to revise the following words:
- enlargement
- reduction
- scale factor

A client asks you to make the following adjustments to the house plan.

1. Enlarge the following by scale factor 2.
   a. Garage
   b. Bathroom 3
2. Join bedrooms 1 and 2 and reduce by scale factor 2.

3. Replace bedroom 3 with a bathroom the same size as bathroom 1.

4. Enlarge the TV room into a very large entertainment room by scale factor 3.

5. Double the size of the study.

6. Enlarge the swimming pool by scale factor 2.

Note: you may want to change the orientation of the enlarged or reduced rooms.
1. Make the following geometric objects using the nets below. Enlarge the nets by a scale factor of 2. You will need some grid paper, a ruler, sticky tape and a pair of scissors.

What kind of prisms are these?

What kind of pyramids are these?

a. d. e.

b. c.

e.
2. Identify and name all the geometric solids (3-D objects) in these diagrams.
   a.  
   b.  
   c.  

3. Identify, name and label as many pyramids and prisms as you can in these photos.
   a.  
   b.  
   c.  

4. Compare prisms and pyramids.

<table>
<thead>
<tr>
<th>Prisms</th>
<th>Pyramids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Problem solving**

Name five pairs of a pyramid and a prism that will exactly fit on top of each other, and say why.
1. Which pyramid will fit exactly onto each prism? Draw lines to match them.

a. Circle the tetrahedron in blue.
b. Circle the hexahedron in red.

2. Describe the prisms and pyramids in these pictures.

a.  

b.  

c.  
3. Your friend made this drawing of a building she saw. Identify and name the solids.

4. Draw the nets for the following:

- Tetrahedron
- Hexahedron

**Problem solving**

- How many tetrahedrons do you need to complete the big tetrahedron?
- How would you use the word hexahedron to describe this Rubic cube?
1. Which geometric solid can be made with these geometric figures?

   a. [Hexagonal prism]
   b. [Triangular prism]
   c. [Octagonal prism]

2. Identify all the geometric figures in these solids and make a drawing of all the shapes.

   a. [Triangular prism]
   b. [Octagonal prism]

3. a. Use waste products to make these geometric solids:
   - prisms (triangular prism, cube, rectangular, pentagonal, hexagonal and octagonal)
   - pyramids (triangular, tetrahedron, rectangular, pentagonal, hexagonal and octagonal)

   b. Use the geometric solids to create “buildings of the future”.

A 2-D shape is a "geometric figure" and a 3-D object is a "geometric solid".
4. a. Write down how you created each polyhedron, focusing on the shapes of the faces and how you joined them. You may include drawings.

b. Write a description of how you put the geometric solids together to create your “buildings of the future”. Say why you use certain solids for certain buildings.

c. Present your work to the class.

Presentation tips

When presenting you should:
- Make eye contact with different people throughout the presentation
- Start by explaining what the presentation is about
- Use natural hand gestures to demonstrate
- Stand up straight with both feet firmly on the ground
- Demonstrate a strong positive feeling about the topic during the entire presentation
- Stay within the required time
- Use visual aids to enhance the presentation
- Explain all points thoroughly
- Organise your presentation well and maintain the interest level of the audience

Problem solving

Fit two geometric solids on top of each other. Where they touch the faces should be the same. The two geometric solids cannot be prisms or pyramids.
**Visualising 3-D objects/playing a game**

**What geometric solid is it?**

1. Ask your friend to close his or her eyes. Then ask him or her the following questions:

   a. Name and describe the **new solid**.

      - Imagine you have a cube.
      - Imagine you now have two identical cubes.
      - Place them together.

   After imagining the object, draw, name and describe it.

<table>
<thead>
<tr>
<th>Draw:</th>
<th>Name:</th>
<th>Describe:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Name and describe the solid from **different views**.

      - Imagine you are looking at a large cardboard box that looks like a cube.
      - Can you stand so that you can see only one square?
      - Can you stand so that you can see 2 or 3 squares?

   Seeing one square
   Seeing two squares
   Seeing three squares
The pyramids are the stone tombs of the Pharaohs – the kings of ancient Egypt. They have stood for thousands of years, filled with many clues about what life (and death) was like in ancient Egypt.

**What is the great pyramid of Giza? Find out.**

*Great pyramid of Giza and maths.*
- The base originally measured about 230.33 m square.
- The original height was 146.59 m.
- A total of over 2,300,000 stone blocks of limestone and granite were used.
- The construction date was about 2589 B.C.
- Estimated construction time was 20 years.
- Estimated total weight is 6.5 million tons.

**c.** What type of pyramid (geometric objects) are we mostly likely to find in Egypt?

**d.** Name and describe the solid from different views.

Imagine you are visiting the pyramids in Egypt.
You are standing on the ground, looking at a pyramid.
What is the maximum number of triangles you see?
What if you were in an aeroplane flying overhead?

**e.** Name and describe the solid from different views.

View from the ground
Aerial view

An aerial view is also called a bird’s eye view. Why do you think it has this name?

**Problem solving**

Describe a geometric solid to your family and ask them to imagine it.
1. Label the following using the words: face, edge and vertex.

a. 

b. 

c. 

d. Mark the apex on each building with a star (*).

2. Label the faces, vertices and edges on each photograph.

a. 

b. 

c. 

An apex is the highest point of a geometric solid with respect to a line or plane chosen as base.
3. What do these objects have in common? When closed, they all have:

a. 

- ___ faces
- ___ edges
- ___ vertices

b. 

- ___ faces
- ___ edges
- ___ vertices

c. 

- ___ faces
- ___ edges
- ___ vertices

d. 

- ___ faces
- ___ edges
- ___ vertices

4. Label the following using the words: surface (face), edge and vertex. Also say which geometric object each one will form.

a. 

Geometric object: 

- ___ edges
- ___ vertices
- ___ faces

b. 

Geometric object: 

- ___ edges
- ___ vertices
- ___ faces

c. 

Geometric object: 

- ___ edges
- ___ vertices
- ___ faces

d. 

Geometric object: 

- ___ edges
- ___ vertices
- ___ faces

5. Look at these skeletons. Say how many vertices and edges you see in each structure

a. 

___ vertices ___ edges

b. 

___ vertices ___ edges

c. 

___ vertices ___ edges

d. 

___ vertices ___ edges

e. 

___ vertices ___ edges

f. 

___ vertices ___ edges

Problem solving

- Can a prism have an odd number of vertices? Give an example.
- Can a pyramid have an odd number of vertices?
- How many more faces does an octagonal pyramid have than a heptagonal pyramid?
Think! Look at these nets of geometric solids. How many faces, vertices and edges does each solid have?

1. Write labels with arrows pointing to the geometrical figures which you can see in each object, and write down how many of each the object contains.

Identify all the geometric figures in this geometric solid. We provide you with four views of the geometric solid to help you.
a. Look at the table above and compare a triangular pyramid and a square pyramid. Describe the similarities and differences between them.

b. Describe the differences between a hexagonal prism and an octagonal prism.

c. Describe the differences between a hexagonal pyramid and an octagonal pyramid.

d. What should you do to the geometric solid on the left to change it to the geometric solid on the right?

i. 

ii. 

Solve this with a family member.

Describe the geometric solid using the words surfaces (faces), vertices and edges. We give you the unfoldings to help you to solve this.
Even more faces, edges and vertices

Revise the following:
- surfaces (faces)
- vertices
- edges

1. Look at the different polyhedra.

Identify the surfaces (faces), vertices and edges.

2. Visualise how many vertices a pentagonal prism has. ___
   a. How many edges does it have? ____
   b. How many faces? ____
   c. What about a heptagonal prism? ____
   d. Or a heptagonal pyramid? ____
### Problem solving

Look at Euler’s formula. This equation shows us the number of vertices, faces, and edges: $8 - 7 + 1 = 2$. Is this a polyhedron? Why or why not?

### Complete the table

<table>
<thead>
<tr>
<th>Solid</th>
<th>Vertices</th>
<th>Edges</th>
<th>Faces</th>
<th>Calculate $F - E + V$ for each geometric solid. $F = $ faces, $E = $ edges and $V = $ vertices. What do you notice?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular prism</td>
<td>6</td>
<td>9</td>
<td>5</td>
<td>$5 - 9 + 6 = 2$</td>
</tr>
<tr>
<td>Rectangular prism</td>
<td>8</td>
<td>12</td>
<td>6</td>
<td>$6 - 12 + 8 = 2$</td>
</tr>
<tr>
<td>Pentagonal prism</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagonal prism</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Octagonal prism</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular pyramid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square pyramid</td>
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<td></td>
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<tr>
<td>Pentagonal pyramid</td>
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<td></td>
</tr>
<tr>
<td>Hexagonal pyramid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Octagonal pyramid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Make a cube and put it in front of you. Turn it to look at different views.

How to make a cube:

1. Step 1
2. Step 2
3. Step 3
4. Step 4
1. Look at the drawings below. Explain them.

See if you can draw a cube at an angle of 30° as below in b, without a protractor. Place a cube on your desk and put a piece of paper under the cube.
2. Draw the following step by step:

**Step 1**  
Draw a line parallel to the side of the table. Then draw a line perpendicular to the vertex that touches the line.

**Step 2**  
Place the cube on the line in the way you see it (approximately 30° turned). Trace around the base of the cube.

**Step 3**  
Remove the cube.

**Step 4**  
Measure your angle to see how close your estimation was.

**Step 5**  

- Measure the length of the sides.
- Draw lines showing the height of the cube of the same length.
- Draw the top of the cube.

**Step 6**  
It is important to use dotted lines to show the back of the cube (or any other geometric solid).
Problem solving

Sit at your desk, look at the sketches below and then place the geometric solid in the same position on your desk. Are all of the drawings possible? Make a drawing of any of these solids showing it in four steps. Remember to make the lines of the back view dotted.
Constructing a pyramid net

What is a pyramid? Look at the pictures and describe a pyramid.

Where do we find real pyramids?

Do we find pyramids only in Egypt?

1. Construct the net for a tetrahedron.

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>Step 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct an equilateral triangle. Label it ABC.</td>
<td>Construct another equilateral triangle with one base joined to base AB of the first triangle.</td>
</tr>
<tr>
<td><img src="image1" alt="Step 1 Diagram" /></td>
<td><img src="image2" alt="Step 2 Diagram" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3:</th>
<th>Step 4:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct another triangle using BD as a base.</td>
<td>Construct another triangle using AD as the base.</td>
</tr>
<tr>
<td><img src="image3" alt="Step 3 Diagram" /></td>
<td><img src="image4" alt="Step 4 Diagram" /></td>
</tr>
</tbody>
</table>
2. Construct a square pyramid net.

Step 1:
Construct two perpendicular lines. The lengths of AD and AB should be the same. Use your pair of compasses to measure them. From there, construct square ABCD.

Step 2:
• Using AB as a base, construct a triangle.
• Using DC as a base, construct a triangle.

Step 3:
• Using DA as a base, construct a triangle.
• Using BC as a base, construct a triangle.

i) After you have constructed the square–based pyramid, answer the following questions:
• what difficulties did you have?
_______________________________________________________________________________
_______________________________________________________________________________
• what would you do differently next time?
_______________________________________________________________________________
_______________________________________________________________________________

ii) Now do the construction on cardboard, cut it out and make the square pyramid.
Construct a net of a prism

What is a prism? Look at the pictures and describe a prism.

1. Construct the net of a triangular prism.

**Step 1:**
Construct two perpendicular lines. The lengths of AD and AB could be the same or one could be longer to form a rectangle. Use your pair compasses to measure them. From there, construct square (or rectangle) ABCD.

**Step 2:**
- Using DC as a base, construct a square (or rectangle).
- Using AB as a base, construct another square (or rectangle).

**Step 3:**
- Using DA as a base, construct a triangle.
- Using BC as a base, construct a triangle.

Some people think a prism only takes on this shape. How can you find out if this is true?
2. Construct a rectangular prism.

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>Step 2:</th>
<th>Step 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct two perpendicular lines. The length between A and B should be longer than that between D and A. Use your compass to measure them. From there, construct rectangle ABCD.</td>
<td>Use DC as a base to construct another rectangle above. Use AB as a base to construct another rectangle below. Label the new points G and H. Use GH as a base to construct another rectangle.</td>
<td>Use DA as a base to construct a square. Use CB as a base to construct a square.</td>
</tr>
</tbody>
</table>

Problem solving

What does this prism show us?
“What is the temperature on a hot, sunny day?”

Point out the degrees on this thermometer. What does it mean when the temperature is two degrees below zero? Show where this is on the thermometer.

You would use a negative sign to write this number since it is below zero.

Where is five degrees below zero on the thermometer? Is this hotter or colder than two degrees below zero?

If you turn the thermometer sideways it looks like a number line and now you can see that the negative numbers are to the left of zero and the positive numbers are to the right of zero, with zero being neither positive nor negative.

1. Write the appropriate temperature for the stated weather condition.
   a. What would the temperature be on a hot and sunny day? _________________
   b. What would the temperature be on a cool spring day? _________________
   c. What would the temperature be on a frosty winter morning? _________________
   d. Write the temperature of eight below zero. _________________________
   e. Which is colder, eight below zero or 10 below zero? Why? _________________
   f. Draw a thermometer and label where 10 below zero would be.

2. Where will the money mentioned in each sentence go, in the negative or positive column?

<table>
<thead>
<tr>
<th>Statement</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Peter won R100 in the draw.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Peter gave his twin sister half of his prize.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. Cindy lost her purse with R20 in it.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem solving

Take a newspaper and find five negative numbers in it.

a. Explain what each number tells us.
b. Write down the opposite numbers for the five numbers.

3. Complete the questions below after completing the table in Question 2.
   a. Circle the key word in each sentence that helped you to make the decision.
   b. What characteristics are found in the positive column? ____________________________
   c. What characteristics are found in the negative column? ____________________________
   d. Write down all the characteristics of integers. ____________________________
   e. Where are integers used in everyday life? Give examples of your own or cut out examples from a newspaper.

4. Complete these number lines.
   a. -1 0 1
   b. -10 -5 0 5
   c. 0 2 4
   d. -10 0 10
   e. -2 -1 0
   f. -3 0 3

5. Complete the following:
   a. {3, 2, 1, 0, ___, ___, ___}
   b. {-10, -9, -8, ___, ___, ___}
   c. {8, 6, 4, 2, ___, ___, ___, ___}
   d. {-9, -6, -3, ___, ___, ___}
   e. {12, 8, 4, ___, ___, ___}
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More integers

• What do we call the units to the right of the zero?
• What do we call the units to the left of the zero?
• What will five units to the left of 3 be?
• What will five units right of 3 be?
• What is the opposite of –4?
• What is the opposite of 4?
• What is three below zero?

1. Write an integer to represent each description.
   a. Five units to the left of 4 on a number line.
   b. 20 below zero.
   c. The opposite of 271.
   d. Eight units to the left of –3 on a number line.
   e. Eight units to the right of –3 on a number line.
   f. 16 above zero.
   g. 14 units to the right of –2 on a number line.
   h. Seven units to the left of –8 on a number line.
   i. The opposite of –108.
   j. 15 below zero.

2. Order these integers from smallest to biggest.
      ___________________________________________
      b. 42, 21, 48, 72, –64, –20
         ___________________________________________
      c. 15, –30, –14, –3, 9, 31, 21, 26, 4, –31, –24, 44
         ___________________________________________
d. –41, 54, –31, –79, 57

e. –26, 32, 23, 10, –31, 12, 31, 26


g. –41, –23, –31, 40, –21, 2

h. 4, –10, 15, 7, 10, –2, –13, –6, –12, 9, 12

i. –7, –15, –25, –24, –12, –13, 22, 6, 11, 2

j. 73, –24, –20, 21, –44, 5, –2, 41, 55

3. Fill in <, > or =
a. –2 2   b. –10 10   c. –5 0
d. –4 –3   e. –9 –6   f. –20 –16

4. Give five numbers smaller than and five numbers bigger than:
a. –2   b. –99   c. 1

<table>
<thead>
<tr>
<th>Smaller</th>
<th>Bigger</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

Problem solving
Make your own word problem using a negative and a positive number
What is the opposite of \(-3\)? How many units are there from \(-3\) to 3?

![Number line diagram]

Explain the lines above.

1. We have learnt that two integers are opposites if they are the same distance away from zero. Write down the opposite integers for the following:
   a. \(-2\)  
   b. 3  
   c. \(-7\)  
   d. 8  
   e. \(-10\)  
   f. \(-15\)  
   g. 1  
   h. \(-100\)  
   i. 75  

2. Calculate the following.
   Example: \(-4 + 2 = -2\)

   a. \(-5 + 5 =\)  
   b. \(-2 + 3 =\)  
   c. \(-7 + 8 =\)  
   d. 2 – 3 =  
   e. \(+4 – 6 =\)  
   f. 10 – 12 =  
Problem solving

What is:

- The sum of 10 and 8, and the sum of −9 and −8?
- The sum of 101 and 85, and the sum of −98 and −104?
- The sum of 19 and −8, and the sum of −19 and 8?
- The sum of −7 and −14, and the sum of −4 and 20?
- The sum of 100 and −50, and the sum of −100 and 50?
Discuss the following:

**Add integers with the same sign**

Find \(-5 + (-2)\).

**Method 1: Use a number line.**
- Start at zero.
- Move 5 units left.
- From there, move 2 units left.

\(-5 + (-2) = -7\)

**Method 2: Draw a diagram.**

\[-5 + (-2) = -7\]

---

**Add integers with different signs**

Find \(5 + (-7)\).

**Method 1: Use a number line.**
- Start at zero.
- Move 5 units right.
- From there, move 7 units left.

\(5 + (-7) = -2\)

**Method 2: Draw a diagram.**

\[5 + (-7) \quad 5 - 7 = -2\]

---

1. Complete the following.
   - Number line method
   - Drawing a diagram

   a. Find \(-8 + (-3)\)

   b. Find \(-12 + (-8)\)

   c. Find \(-4 + (-5)\)

   d. Find \(-7 + (-9)\)

   e. Find \(-18 + (-7)\)

   f. Find \(6 + (-8)\)
g. Find $9 + (-11)$

h. Find $6 + (-9)$

i. Find $3 + (-16)$

j. Find $8 + (-19)$

2. Write sums for the following.

a. 

b. 

c. 

d. 

e. 

f. 

g. 

Help a friend!

Write down step-by-step how you would explain integer operations to a friend who missed a day at school.
Adding and subtracting integers

Subtracting a negative number is just like adding a positive number. The two negatives cancel each other out. \( 2 + 3 = 2 - (-3) \)

If you are \textbf{adding} a \textbf{positive number}, move your finger to the \textbf{right} as many places as the value of that number. For example, if you are adding 3, move your finger three places to the right: \( 2 + 3 = 5 \)

If you are \textbf{subtracted} a \textbf{negative number}, move your finger to the right as many places as the value of that number. For example, if you are subtracting \(-3\), move your finger three places to the right: \( 2 - (-3) = 5 \)

Adding a negative number is just like subtracting a positive number: \( 2 + (-3) = 2 - 3 \)

If you are \textbf{adding} a \textbf{negative number}, move your finger to the \textbf{left} as many places as the value of that number. For example, if you are adding \(-3\), move your finger three places to the left: \( 2 + (-3) = -1 \)

If you are \textbf{subtracting} a \textbf{positive number}, move your finger to the \textbf{left} as many places as the value of that number. For example, if you are subtracting 3, move your finger three places to the left: \( 2 - 3 = -1 \)

1. Calculate the following, using number lines:
   a. \( 4 + (-5) = \)  

   \[ \text{ } \]

   b. \( 5 + (-7) = \)  

   \[ \text{ } \]

   c. \( 7 + (-8) = \)  

   \[ \text{ } \]
Problem solving

Make your own problem using integers.

d. $6 + (-9) =$


e. $3 + (-2) =$

f. $4 + (-7) =$

2. Calculate the following:
   a. $4 - (-5) =$
   b. $5 - (-7) =$
   c. $5 - (-7) =$
   d. $6 - (-9) =$
   e. $3 - (-2) =$
   f. $4 - (-7) =$
   g. $5 - (-4) =$
   h. $2 - (-1) =$
   i. $3 - (-4) =$
   j. $1 - (-3) =$
   k. $2 - (-5) =$
   l. $5 - (-11) =$
   m. $7 - (-6) =$
   n. $8 - (-12) =$
   o. $5 - (-9) =$
   p. $4 - (-4) =$
   q. $3 - (-3) =$
   r. $5 - (-12) =$
   s. $2 - (-4) =$
   t. $3 - (-6) =$
   u. $5 - (-6) =$
   v. $3 - (-8) =$
   w. $7 - (-10) =$
   x. $6 - (-6) =$
   y. $4 - (-6) =$
   z. $7 - (-14) =$

3. Explain in your own words what you had to do to get to the answer.
a. In number 1.

b. In number 2.
## Integer Calculations

### Describe:

<table>
<thead>
<tr>
<th>Positive number</th>
<th>Negative number</th>
<th>=</th>
<th>Positive answer</th>
<th>Negative answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive number</td>
<td>Negative number</td>
<td>=</td>
<td>Positive answer</td>
<td>Negative answer</td>
</tr>
<tr>
<td>Negative number</td>
<td>Positive number</td>
<td>=</td>
<td>Positive answer</td>
<td>Negative answer</td>
</tr>
<tr>
<td>Negative number</td>
<td>Positive number</td>
<td>=</td>
<td>Positive answer</td>
<td>Negative answer</td>
</tr>
</tbody>
</table>

### Give an example of each using symbols:

1. **Calculate the following:**
   
   a. $12 + (-31) =$
   
   b. $(-28) + (-42) =$
   
   c. $7 + (-34) =$
   
   d. $33 + (-44) =$
   
   e. $5 + (-432) =$
   
   f. $(-15) + (-20) =$
   
   g. $(-15) + 5 =$
   
   h. $19 + 14 =$
   
   i. $25 + 4 =$
   
   j. $4 + 7 =$

2. **Calculate the following.**

   Example: $-14 - (-20)$
   
   $= -14 + 20$
   
   $= 6$
1. Sign:
Date:

Problem solving

Give three integers of which the sum is –9. Use two positive integers and one negative integer.

a. 7 – (–31) = 

d. 47 – (–46) = 

g. (–47) – (–7) = 

j. 5 – 31 =

3. Solve the following:

a. _____ + 44 = 42
b. _____ + (–18) = –32
c. _____ + (–21) = –30

d. (–3) + _____ = 33
e. 14 + _____ = 16
f. 14 + _____ = 63

g. 42 + _____ = 65
h. _____ + (–10) = –12
i. 38 + _____ = 65

j. (–46) + _____ = –72
k. _____ + (–43) = –41
l. _____ + (–16) = 30

m. _____ + (–44) = –81
n. _____ + (–31) = 6
o. _____ + (–28) = –32

p. 11 + _____ = –19
q. _____ + 24 = 6
r. 45 + _____ = 73

s. _____ + (–29) = 1
t. 12 + _____ = –32
u. (–44) + _____ = –15

v. _____ + 24 = –11
w. _____ + 10 = 33
x. _____ + 49 = 18

y. _____ + 4 = 26
z. 41 + _____ = 60

Problem solving

Give three integers of which the sum is –4. Use two negative integers and one positive integer.

Give four integers of which the sum is –11. Use two negative integers and two positive integers.
1. Use the commutative property to change the following expressions to equations.

Example: \(8 + (-3) = (-3) + 8 = 5\)
\((-8) + 3 = 3 + (-8) = -5\)

a. \(4 + (-5)\)  
b. \((-10) + 7\)  
c. \(3 + (-9)\)

d. \(8 + (-11)\)  
e. \((-4) + 8\)  
f. \(9 + (-2)\)

2. Show that the commutative property holds for the addition of integers.

Example: \(a = -2\) and \(b = 3\)
\(a + b = b + a\)
\((-2) + 3 = 3 + (-2)\)
\(1 = 1\)

a. \(a + b = b + a\) if \(a = 4; b = -1\)  
b. \(a + b = b + a\) if \(a = -2; b = 7\)
Problem solving

Use the commutative property to make your own equation and prove that it is satisfied using the numbers -8 and 21.

c. \(a + b = b + a\) if \(a = -2; b = 7\)

d. \(x + y = y + x\) if \(x = -1; y = 13\)

e. \(x + y = y + x\) if \(x = -5; y = 9\)

f. \(d + e = e + d\) if \(e = -12; d = 7\)

g. \(t + s = s + t\) if \(t = -4; s = 10\)

h. \(a + b = b + a\) if \(a = -10; b = 7\)

i. \(y + z = z + y\) if \(z = -8; y = 2\)

j. \(k + m = m + k\) if \(k = -13; m = 20\)
**The Associative property of numbers** means that it doesn’t matter how you **group the numbers** when you **add** or when you **multiply**.

**Example addition:**

\[(2 + 3) + 5 = 2 + (3 + 5)\]

Because \(5 + 5 = 2 + 8 = 10\)

**Example multiplication:**

\[(2 \times 4) \times 3 = 2 \times (4 \times 3)\]

\[8 \times 3 = 2 \times 12 = 24\]

So, in other words it doesn’t matter which you calculate first.

In this worksheet we will look at integers and the associative property.

1. Use the associative property to calculate the following.

   **Example:**

   \[
   \begin{align*}
   &[(2 + 3) + (-4)] = 2 + [3 + (-4)] \\
   &5 - 4 = 2 - 1 \\
   &1 = 1 \\
   &\end{align*}
   \]

   \[
   \begin{align*}
   &[-2 + (3 + 4)] = [-2 + 3] + 4 \\
   &-2 + 7 = 1 + 4 \\
   &5 = 5 \\
   &\end{align*}
   \]

   \[
   \begin{align*}
   &[-3 + (2 + 4)] = [(-3 + 2) + 4] \\
   &-3 + 6 = -1 + 4 \\
   &3 = 3 \\
   &\end{align*}
   \]

   **a.** \([(-6) + (4 + 2)]\)

   **b.** \([3 + 7 + (-5)]\)

   **c.** \([(6 + 4) + (-2)]\)

   **d.** \([(-3) + 7 + 5]\)

   **e.** \([(-4) + (6 + 2)]\)

   **f.** \([3 + (-7) + 5]\)

   **g.** \([(-9) + (3 + 11)]\)

   **h.** \([(12 + 13) + (-10)]\)

   **i.** \([(3 + 9) + (11)]\)

   **j.** \([(12) + (13 + 10)]\)
2. Show that the associative property for addition holds for integers.

<table>
<thead>
<tr>
<th>Example:</th>
<th>( a = -7, \ b = 1, \ c = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a + b) + c = a + (b + c))</td>
<td></td>
</tr>
<tr>
<td>([-7 + 1] + 2 = (-7) + (1 + 2))</td>
<td></td>
</tr>
<tr>
<td>(-6 + 2 = -7 + 3)</td>
<td></td>
</tr>
<tr>
<td>(-4 = -4)</td>
<td></td>
</tr>
</tbody>
</table>

| a. \((a + b) + c = a + (b + c)\) |
|---|---|
| If: \( a = 4 \) |
| \( b = -5 \) |
| \( c = 3 \) |

| b. \((a + b) + c = a + (b + c)\) |
|---|---|
| If: \( a = 2 \) |
| \( b = 9 \) |
| \( c = -4 \) |

| c. \(a + (b + c) = (a + b) + c\) |
|---|---|
| If: \( a = -8 \) |
| \( b = 1 \) |
| \( c = 2 \) |

| d. \(a + (b + c) = (a + b) + c\) |
|---|---|
| If: \( a = -2 \) |
| \( b = 11 \) |
| \( c = 12 \) |

**Problem solving**

Use the associative property to make your own equation and prove that it is equal using the numbers –5, 17 and 12.
Integers: distributive property and integers

The distributive property of number says you get the same answer when you ... I cannot remember, please help me.

...multiply a number by a group of numbers added together as when you do when you multiply each number separately and then add the products.

Oh! So the 4 × can be distributed across the 2 + 5.

In this worksheet we will work with integers.

1. Use the distributive property to calculate the sums. Before you calculate, highlight or underline the distributed number.

Example:

\[ -2 \times (3 + 4) = (-2 \times 3) + (-2 \times 4) \]
\[ -2 \times 7 = -6 + -8 \]
\[ -14 = -14 \]

\[ 2 \times (-3 + 4) = (2 \times -3) + (2 \times 4) \]
\[ 2 \times 1 = -6 + 8 \]
\[ 2 = 2 \]

\[ 2 \times (3 + -4) = (2 \times 3) + (2 \times -4) \]
\[ 2 \times (-1) = 6 + -8 \]
\[ -2 = -2 \]

a. \(-4 \times (2 + 1)\)

b. \(-5 \times (3 + 6)\)

c. \(4 \times (-2 + 1)\)

d. \(5 \times (-3 + 6)\)

e. \(4 \times (2 + -1)\)

f. \(5 \times (3 + -6)\)

g. \((-3 \times 2) + (-3 \times 4)\)

h. \((-7 \times 1) + (-7 \times 4)\)

i. \((8 \times -4) + (8 \times 2)\)
2. Substitute and calculate.

Example: \( a \times (b + c) \) if \( a = -4, b = 3, c = 1 \)

\[
a \times (b + c) = (a \times b) + (a \times c)
-4 \times (3 + 1) = (-4 \times 3) + (-4 \times 1)
-4 \times 4 = -12 + -4
-16 = -16
\]

a. \( a \times (b + c) \)
if \( a = 2, b = -3, c = -5 \)

b. \( a \times (b + c) \)
if \( a = -7, b = 2, c = 3 \)

c. \( a \times (b + c) \)
if \( a = 1, b = -8, c = 2 \)

d. \((a \times b) + a + c)\)
if \( a = 3, b = -10, c = 5 \)

e. \( m \times (n + p) \)
if \( m = 3, n = 2, p = -11 \)

f. \((m \times n) + (m \times p)\)
if \( m = 7, n = 8, p = -9 \)

Problem solving

Make use of the distributive property to write your own equation for:
\( a = -4, b = 5 \) and \( c = 11 \)
Number patterns: constant difference and ratio

Describe the patterns using "adding" and "subtracting".

- Adding 4: –12, –8, –4, 0
- Subtracting 5: 1, –4, –9, –14

1. Describe each pattern.

a.

b.

c.

d.

Describe the pattern in your own words.
Brenda collects shells. Every day she picks up double the amount of the previous day. On day 1 she picks up 8 shells. On day 2 she collects 16. How many shells will she pick up on day 3 if the pattern continues? Write down the rule.
115 Number patterns: neither a constant difference nor a constant ratio

Describe the following: –1, –2, –4, –7, –11, –16, ...
What will the next three terms be, using the identified rule?
This pattern has neither a constant difference nor a constant ratio. It can be described in your own words as “decreasing the difference between consecutive terms by 1 each time” or “subtracting 1 more than what was subtracted to get the previous term”. Using this rule, the next three terms will be -22, -29, -37.

1. Describe the pattern and make a drawing to show the value of each term.

Example: 15, 22, 16, 21, 17

<table>
<thead>
<tr>
<th>Term 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. –4, 1, 5, 8, 10</td>
</tr>
<tr>
<td>b. 8, 10, 13, 17, 22</td>
</tr>
<tr>
<td>c. 2, –2, –8, –16, –26</td>
</tr>
<tr>
<td>d. –11, –12, –10, –13, –9</td>
</tr>
<tr>
<td>e. –7, –1, 11, 29, 53</td>
</tr>
<tr>
<td>f. 5, –3, –10, –16, –21</td>
</tr>
</tbody>
</table>

Take your time to describe the pattern in words.

2. What will the value of the tenth pattern be?

Example: 2, 3, 5, 8, 12
Add +1, +2, +3, +4

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a. Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b. Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c. Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>–5</td>
<td>–2</td>
<td>4</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>
111

Sign:

Date:

Problem solving

Thabo builds a brick wall around the perimeter of his house. On the first day he uses 75 bricks, on the second day he uses 125 and on the third day he uses 175. How many bricks will he need on the fourth day?

Write a rule for the pattern.

3. What will the value of the term be? Complete the table.

Example: 1, 4, 9, 16

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>-6</td>
<td>0</td>
<td>12</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

d. Position in the sequence | 1 | 2 | 3 | 4 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>4</td>
<td>16</td>
<td>128</td>
<td></td>
</tr>
</tbody>
</table>

e. Position in the sequence | 1 | 2 | 3 | 4 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>4</td>
<td>16</td>
<td>128</td>
<td></td>
</tr>
</tbody>
</table>

3. What will the value of the term be? Complete the table.

Example: 1, 4, 9, 16

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>225</td>
</tr>
</tbody>
</table>

a. Position in the sequence | 1 | 2 | 3 | 4 | 20 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

b. Position in the sequence | 1 | 2 | 3 | 4 | 104 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>-4</td>
<td>-8</td>
<td>-12</td>
<td>-16</td>
<td></td>
</tr>
</tbody>
</table>

c. Position in the sequence | 1 | 2 | 3 | 4 | 59 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td></td>
</tr>
</tbody>
</table>

d. Position in the sequence | 1 | 2 | 3 | 4 | 36 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>13</td>
<td>26</td>
<td>39</td>
<td>52</td>
<td></td>
</tr>
</tbody>
</table>

e. Position in the sequence | 1 | 2 | 3 | 4 | 29 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>21</td>
<td>42</td>
<td>63</td>
<td>84</td>
<td></td>
</tr>
</tbody>
</table>

Problem solving

Thabo builds a brick wall around the perimeter of his house. On the first day he uses 75 bricks, on the second day he uses 125 and on the third day he uses 175. How many bricks will he need on the fourth day? Write a rule for the pattern.

Ravi draws 2 figures on the first page, 4 figures on the second page, 8 figures on the third page, and 16 figures on the fourth page. If this pattern continues, how many figures will Ravi draw on the fifth page?

Lisa read 56 pages on Sunday, 66 pages on Monday, 76 pages on Tuesday, and 86 pages on Wednesday. If this pattern continued, how many pages would Lisa read on Thursday?

Thandi cut 1 rose flower from the first plant, 3 roses from the second plant, 7 roses from the third plant, and 13 roses from the fourth plant. If this pattern continued, how many rose flowers would Thandi cut from the fifth plant?
Describe the relationships between the numbers in a sequence. 
\(-4, -7, -10, -13, \ldots\)

**Identify the:**
- First term: \(-4\)
- Second term: \(-7\)
- Third term: \(-10\)
- Fourth term: \(-13\)

**What are the rules for the sequences:** ("subtracting 3")
- First term: \(-4 = -3(1) - 1\)
- Second term: \(-7 = -3(2) - 1\)
- Third term: \(-10 = -3(3) - 1\)
- Fourth term: \(-13 = -3(4) - 1\)

If the number in the brackets represents the term, what will the 20th term be?

**1. Look at the following sequences:**

   i. Calculate the 20th term.
   ii. Describe the rule in your own words.

**Example:** Number sequence: \(-6, -10, -14, -18\)
- Rule in words: \((-4 \times \text{the position of the term}) - 2\).

a. Number sequence: \(8, 14, 20, 26\)
   i. 
   ii. 

b. Number sequence: \(0, -3, -6, -9\)
   i. 
   ii. 

c. Number sequence: \(-4, -5, -6, -7\)
   i. 
   ii. 

d. Number sequence: \(-2, 3, 8, 13\)
   i. 
   ii.
Problem solving

Tshepo earns R25 per week for washing his father’s motor car. If he saves R5.50 the first week, R7.50 the second week and R9.50 the third week, how much will he save in the fourth week if the pattern continues?

Calculate the total amount he saves over 4 weeks. Write a rule for the number sequence.
A sequence is a list of numbers or objects which are in a special order.

Example:

Arithmetic number sequence: –2, –4, –6, –8
Geometric number sequence: –2, –4, –8, –16

What is the difference between an arithmetic number sequence and a geometric number sequence? Give one example of each.

1. Describe the sequence in different ways using the template provided.

Example: –6, –13, –20, –27

i) Write it on a number line.

ii) Write it in a table.

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>–6</td>
<td>–13</td>
<td>–20</td>
<td>–27</td>
</tr>
</tbody>
</table>

iii) Where $n$ is the position of the term.

First term: $-7 \cdot 1 + 1 = -6$
Second term: $-7 \cdot 2 + 1 = -13$
Third term: $-7 \cdot 3 + 1 = -20$
Fourth term: $-7 \cdot 4 + 1 = -27$

$n^{th}$ term: $-7(n) + 1$
### a. \(-1, 2, 5, 8\)

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

iii) Where \( n \) is the position of the term.

First term: 

Second term: 

Third term: 

Fourth term: 

### b. \(3, 5, 7, 9\)

<table>
<thead>
<tr>
<th>Position in the sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

iii) Where \( n \) is the position of the term.

First term: 

Second term: 

Third term: 

Fourth term: 

continued
Number sequences: describe a pattern continued

c. –11, –19, –27, –35

i) 

ii) Position in the sequence  1  2  3  4
  Term

iii) Where n is the position of the term.

First term: 
Second term: 
Third term: 
Fourth term:

$\text{n}^{\text{th}}\text{ term:}$

d. 16, 22, 28, 34

i) 

ii) Position in the sequence  1  2  3  4
  Term


Problem solving

Write the rule for the number sequence: –3, –5, –7, –9

First term:

Second term:

Third term:

Fourth term:

n^th\ term:

iii) Where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:

\( n^{th} \) term:

e. –4, –9, –14, –19

Position in the sequence | 1 | 2 | 3 | 4 |
---|---|---|---|---|
Term | | | | |

iii) Where n is the position of the term.

First term:

Second term:

Third term:

Fourth term:

\( n^{th} \) term:
Look and discuss.

\[ b = -a \times 9 \]

Which numbers can replace \( a \)?
- \( b = -1 \times 9 = -9 \)
- \( b = -9 \times 9 = -81 \)
- \( b = -11 \times 9 = -99 \)
- \( b = -25 \times 9 = -225 \)
- \( b = -8 \times 9 = -72 \)

Calculate:
- \( t = -2 \times 5 + 6 = -16 \)
- \( t = -8 \times 5 + 6 = -46 \)
- \( t = -6 \times 5 + 6 = -36 \)
- \( t = -5 \times 5 + 6 = -31 \)
- \( t = -3 \times 5 + 6 = -21 \)

1. Revision: complete the flow diagrams.

\( a \)

\( b \)

\( p \)

\( t \)

\( p \)

\( t \)

\( p \)

\( t \)

\( t = -p \times -3 \)
2. Use the given rule to calculate the value of b.

Example: \[ a = \begin{array}{c}
3 \\
2 \\
5 \\
7 \\
4 \\
\end{array} \]

\[ b = a \times 4 \]

- \(-3 \times 4 = -12\)
- \(-2 \times 4 = -8\)
- \(-5 \times 4 = -20\)
- \(-7 \times 4 = -28\)
- \(-4 \times 4 = -16\)

a. \[ a = \begin{array}{c}
6 \\
15 \\
8 \\
2 \\
17 \\
\end{array} \]

\[ b = -a \times 6 \]

b. \[ a = \begin{array}{c}
2 \\
8 \\
12 \\
20 \\
29 \\
\end{array} \]

\[ b = a \times 15 \]

c. \[ x = \begin{array}{c}
2 \\
1 \\
3 \\
11 \\
25 \\
\end{array} \]

\[ y = -x + 9 \]

d. \[ r = \begin{array}{c}
15 \\
18 \\
20 \\
31 \\
16 \\
\end{array} \]

\[ s = r + 11 \]
3. Use the given rule to calculate the variable.

Example: 

\[ b = -a \times 2 + 3 \]

a. 
\[ a = b \times 3 + 10 \]

b. 
\[ g = h \times 2 + 15 \]

c. 
\[ x = y \times 2 + 4 \]

d. 
\[ m = n + 8 \times 2 \]
4. Prepare one flow diagram to present to the class.

Problem solving

- Draw your own flow diagram where $a = -c - 9$.
- Draw your own flow diagram where $a = c \times 3 - 7$
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More input and output values

Term 4

1. Solve for \( m \) and \( n \).
   a. \( x = 3y - 1 \)
   
   \[
   \begin{array}{c|c|c|c|c|c}
   x & 1 & 2 & 3 & 4 & 12 \\
   y & 5 & 7 & 9 & 11 & m \\
   \end{array}
   \]
   b. \( x = -2y + 6 \)
   
   \[
   \begin{array}{c|c|c|c|c|c|c|c}
   y & 2 & 4 & 6 & 10 & 20 \\
   x & 1 & 2 & 3 & 5 & n \\
   \end{array}
   \]
   c. \( y = -4x - 2 \)
   
   \[
   \begin{array}{c|c|c|c|c|c|c|c}
   x & 3 & 4 & 5 & 6 & 10 & 100 \\
   y & 23 & 6 & n & 10 & 100 \\
   \end{array}
   \]
   d. \( y = x + 2 \)
   
   \[
   \begin{array}{c|c|c|c|c|c|c|c}
   x & 2 & 4 & 5 & 16 & 17 \\
   y & 5 & m \\
   \end{array}
   \]
   e. \( t = -8s + 2 \)
   
   \[
   \begin{array}{c|c|c|c|c|c|c|c}
   s & 1 & 2 & 3 & 5 & 6 & 7 \\
   t & 30 & m \\
   \end{array}
   \]
   f. \( q = 7p - 7 \)
   
   \[
   \begin{array}{c|c|c|c|c|c|c|c|c|c|c}
   f & 1 & 5 & 10 & 20 & n & 100 \\
   q & m & 168 \\
   \end{array}
   \]

2. What is the value of \( m \) and \( n \)?

Example: \( y = -7x + 2 \)
   
   \[
   \begin{array}{c|c|c|c|c|c|c|c|c|c|c}
   x & 1 & 2 & 3 & 4 & 15 & m & 60 \\
   y & -5 & -12 & -19 & -26 & -103 & 18 & n \\
   \end{array}
   \]
   Rule: the given term \( x \times 7 + 2 \)
   
   \[
   n = -418 \text{ and } m = -2
   \]
Problem solving

What is the tenth term?  4x – 5, 5x – 5, 6x – 5
If y = 5x – 8 and x = 2, 3, 4, ..., draw a table to show it.
Compare the two examples.

\[-5 + 4\]

\[-5 + 4\] is an **algebraic expression**

\[-5 + 4 = -1\]

\[-5 + 4 = -1\] is an **algebraic equation**

1. Say whether it is an expression or an equation.

   | a. \(-4 + 8\) | b. \(-9 + 7 = -2\) | c. \(-5 + 10\) |
   | d. \(-8 + 4 = -4\) | e. \(-7 + 5\) | f. \(-15 + 5 - 10\) |

2. Describe the following:

   **Example:** \(-6 + 2 = -4\)
   - \(-6 + 2\) is an expression that is equal to the value on the right-hand side, \(-4\).
   - \(-6 + 2 = -4\) is called an equation. The left-hand side of an equation equals the right hand side.

   | a. \(-8 + 2 = -6\) | b. \(-15 + 9 = -6\) |
   | c. \(-11 + 9 = -2\) | d. \(-5 + 3 = -2\) |
   | e. \(-8 + 1 = -7\) | f. \(-4 + 3 = -1\) |
3. Use of the variable “a” as well as integers to create 10 expressions of your own.

Example: 5 + a

4. Use of the variable “a” as well as integers to create 10 equations of your own.

Example: 5 + a = 13

5. Say whether it is an expression or an equation.

Example: –8 + a (It is an expression.)
–8 + a = –11 (It is an equation.)

a. –9 + a = –2
b. –3 + a = –1
c. –5 + a = –3
d. –18 + a
e. –12 + a = –3
f. –7 + a

Problem solving

Create 10 examples of algebraic expressions with a variable and a constant. From these create algebraic equations and solve them.
The rule as an expression

The rule is \(-2(n) + 1\)

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>-1</td>
<td>-3</td>
<td>-5</td>
<td>-7</td>
<td>-9</td>
<td>(n)</td>
</tr>
</tbody>
</table>

Write the rule as an expression.

First term: \(-2(1) + 1 = -2 + 1 = -1\)
Second term: \(-2(2) + 1 = -4 + 1 = -3\)
Third term: \(-2(3) + 1 = -6 + 1 = -5\)
Fourth term: \(-2(4) + 1 = -8 + 1 = -7\)
Fifth term: \(-2(5) + 1 = -10 + 1 = -9\)
\(n^{\text{th}}\) term: \(-2(n) + 1 =\)

Note: These expressions all have the same meaning:

- \(-2n + 1\)
- \(-2 \times n + 1\)
- \(-2n + 1\)

1. Describe the following in words:

Example: \(-4, -8, -12, -16, -20, \ldots\)
subtracting 4 from the previous term.

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>-4</td>
<td>-8</td>
<td>-12</td>
<td>-16</td>
<td>-20</td>
<td>(-3(n) - 1)</td>
</tr>
</tbody>
</table>

First term is \(-4(1) - 1\), therefore the rule is \(-4(n) - 1\)

a. 9; 6; 3; 0; -3; ... b. 4; 10; 16; 22; 28; ... c. 7; 14; 21; 28; 35; ...
d. 12; 24; 36; 48; 60; ... e. 8; 16; 24; 32; ... f. 6; 16; 26; 36; 46; ...

d. 12; 24; 36; 48; 60; ...

e. 8; 16; 24; 32; ...

2. Describe the following sequence using an expression:

Example: \(-4, -8, -12, -16, -20, \ldots\)

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>-4</td>
<td>-8</td>
<td>-12</td>
<td>-16</td>
<td>-20</td>
<td>(-3(n) - 1)</td>
</tr>
</tbody>
</table>

First term is \(-4(1) - 1\), therefore the rule is \(-4(n) - 1\)

a. 6; 8; 10; 12; 14 b. 5; 11; 17; 23; 29 c. 4; 13; 22; 31; 40
3. What does the rule mean?

Example: For the following number sequence the rule \(-2n - 1\) means:

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>-3</td>
<td>-5</td>
<td>-7</td>
<td>-9</td>
<td>-11</td>
<td>(-2n - 1)</td>
</tr>
</tbody>
</table>

(-3 is the first term, -5 is the second term, -7 is the third term, etc.)

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>19</td>
<td>22</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>10</td>
<td>18</td>
<td>26</td>
<td>34</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>7</td>
<td>11</td>
<td>15</td>
<td>19</td>
<td>23</td>
<td>(7n - 5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>8</td>
<td>17</td>
<td>26</td>
<td>35</td>
<td>44</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position in sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>24</td>
<td>37</td>
<td>50</td>
<td>63</td>
<td>76</td>
<td></td>
</tr>
</tbody>
</table>

**Problem solving**

Write a rule for the following:

- **On the first day I spend R15, on the second day I spend R30, on the third day I spend R45. How much money do I spend on the tenth if this pattern continues?**
- **I save R15 in January, R30 in February R45 in March. How much money will I save in September if the pattern continues?**
- **Thabo sells one chocolate on Monday, three chocolates on Tuesday and five on Wednesday. How many chocolates will he sell on Friday if the pattern continues?**
- **A farmer plants 2 rows of maize on the first day, 6 rows on the second day and 11 rows on the third day. How many rows must he plant on the 12th day if the pattern continues.**
- **Bongi spends twenty minutes on the computer on day one, thirty minutes on day two and forty minutes on day three. How much time will she spend on the computer on day nine if the pattern continues?**
Describe the rule of this number sequence in words.

What does the rule $-4n + 1$ mean for the number sequence $-3, -7, -11, -15, -19, ...$ mean?

Write the rule as an expression.

First term: $-4(1) + 1 = -3$
Second term: $-4(2) + 1 = -7$
Third term: $-4(3) + 1 = -11$
Fourth term: $-4(4) + 1 = -15$
Fifth term: $-4(5) + 1 = -19$
n$^\text{th}$ term: $-4(n) + 1$

1. Describe the following in words:

   a. $-3; -12; -21; -30; -39$
   b. $-6; -13; -20; -27; -34$
   c. $-3; -5; -7; -9; -11$
   d. $6; -4; -14; -24; -34$
   e. $-7; -8; -9; -10; -11$
   f. $-8; -12; -16; -20; -24$
   g. $-14; -17; -20; -23; -26$
   h. $-19; -21; -23; -25; -27$
   i. $9; -2; -13; -24; -35$
   j. $-1; -6; -11; -16; -21$

Subtracting 4 from the previous term.
2. Describe the following sequence using an expression:

*Example:* 
-2, -6, -10, -14, -18, ...

First term: \(-4(1) + 2\)

- \(-4(n) + 2\)

a. 2, 4, 6, 8, 10, ...

b. 3, 5, 7, 9, 11, ...

c. -8; -20; -32; -44; -56

d. -13; -17; -21; -25; -29

e. -16; -22; -28; -34; -40

f. 9; -2; -13; -24; -35

g. 4; -4; -12; -20; -28

h. -3; -12; -21; -30; -39

i. -8; -18; -28; -38; -48

j. 6; -1; -8; -15; -22

Problem solving

Write three different rules for each of these:

- 3; -3; -9; -15; -21
- -14; -22; -30; -38; -46
- -23; -30; -37; -44; -51
- 5; 4; 3; 2; 1
- 19; 7; -5; -17; -29
The algebraic equation

Solving equations

Because an equation represents a balanced scale, it can also be manipulated like one.

Initial equation is \( x - 2 = -5 \)

Add 2 to both sides \( x - 2 + 2 = -5 + 2 \)

Answer \( x = -3 \)

1. Solve for \( x \).

Example:
\[
\begin{align*}
  x - 5 &= -9 \\
  x - 5 + 5 &= -9 + 5 \\
  x &= -4
\end{align*}
\]

\[
\begin{array}{llll}
a. \quad x - 12 &= -30 & b. \quad x - 8 &= -14 & c. \quad x - 17 &= -38 \\
      &   &   &   \\
  d. \quad x - 20 &= -55 & e. \quad x - 25 &= -30 & f. \quad x - 18 &= -26 \\
      &   &   &   \\
  g. \quad x - 6 &= -12 & h. \quad x - 34 &= -41 & i. \quad x - 10 &= -20 \\
      &   &   &   \\
  j. \quad x - 25 &= -33 &   &   &   \\
\end{array}
\]

2. Solve for \( x \).

Example:
\[
\begin{align*}
  x + 5 &= -2 \\
  x + 5 - 5 &= -2 - 5 \\
  x &= -7
\end{align*}
\]

\[
\begin{array}{llll}
a. \quad x + 7 &= -5 & b. \quad x + 3 &= -1 & c. \quad x + 15 &= -12 \\
      &   &   &   \\
      &   &   &   \\
\end{array}
\]
3. Solve for $x$.

Example: $x - 4 + 2 = -7$

\[ x - 2 = -7 \]

\[ x - 2 + 2 = -7 + 2 \]

\[ x = -5 \]

---

a. $x - 3 = -15$

b. $x - 7 = -12$

c. $x - 2 = -5$

d. $x - 5 = -15$

e. $x - 12 = -20$

f. $x - 10 = -25$

g. $x - 23 = -34$

h. $x - 2 = -7$

i. $x - 30 = -40$

---

Problem solving

Write an equation for the following and solve it:

Five times a certain number minus four equals ninety-five.
More on the algebraic equation

\(-2x = 30\)

What does \(-2x\) mean?

What is the inverse operation of multiplication?

We need to divide \(-2x\) by \(-2\) to solve for \(x\).

\[
-\frac{2x}{-2} = \frac{30}{-2}
\]

\[x = -15\]

Remember you need to balance the scale. What you do on the one side of the equal sign, you must do on the other side as well.

1. Solve for \(x\).

Example: \(-3x = 12\)

\[
-\frac{3x}{-3} = \frac{12}{-3}
\]

\[x = -4\]

a. \(-5x = 60\)  
b. \(-2x = 24\)  
c. \(-12x = 48\)  
d. \(-7x = 21\)  
e. \(-15x = 60\)  
f. \(-9x = 54\)  
g. \(-5x = 10\)  
h. \(-12x = 36\)  
i. \(-8x = 64\)
2. Solve for x.

Example: 
\[-3x - 2 = 10\]
\[-3x - 2 + 2 = 10 + 2\]
\[-3x = -12\]
\[x = -4\]

a. \[-2x - 5 = 15\]
b. \[-9x - 4 = 32\]
c. \[-3x - 3 = 18\]
d. \[-3x - 2 = 22\]
e. \[-8x - 4 = 12\]
f. \[-20x - 5 = 95\]
g. \[-12x - 5 = 55\]
h. \[-7x - 3 = 25\]
i. \[-2x - 2 = 18\]

Problem solving

Write an equation and solve it.
- Negative two times \(y\) equals negative twelve.
- Negative three times \(a\) equals negative ninety-nine.
- Negative five times \(b\) equals negative sixty.
- Negative four times \(d\) equals forty-four.
- Negative three times \(x\) equals thirty.
- Negative two times \(y\) equals sixty-four.
- Negative nine times \(m\) equals one hundred and eight.
- Negative six times \(a\) equals sixty-six.
- Negative five times \(b\) equals fifteen.
- Negative eight times \(c\) equals forty.
### More algebraic equations

<table>
<thead>
<tr>
<th>Term 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Substitute</strong></td>
</tr>
</tbody>
</table>

#### Example: If \( y - x^2 + 2 \); calculate \( y \) when \( x = -4 \)
- \( y = (-4)^2 + 2 \)
- \( y = 16 + 2 \)
- \( y = 18 \)

<table>
<thead>
<tr>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^2 + 1 )</td>
</tr>
<tr>
<td>( y = (-4)^2 + 2 )</td>
</tr>
<tr>
<td>( y = 16 + 2 )</td>
</tr>
<tr>
<td>( 18 = 18 )</td>
</tr>
</tbody>
</table>

#### a. \( y = x^2 + 3 \); \( x = 3 \)

#### b. \( y = b^2 + 3 \); \( b = 4 \)

#### c. \( y = x^2 + 2 \); \( x = 4 \)

#### d. \( y = q^2 + 9 \); \( q = 5 \)

#### e. \( y = c^2 + 1 \); \( c = 7 \)

#### f. \( y = p^2 + 6 \); \( p = 2 \)

#### g. \( y = d^2 + 7 \); \( d = 9 \)

#### h. \( y = x^2 + 5 \); \( x = 3 \)

#### i. \( y = f^2 + 8 \); \( f = 10 \)

#### j. \( y = x^2 + 4 \); \( x = 12 \)
2. Substitute and calculate.

Example: If \( y = x^2 + \frac{2}{x} \); calculate \( y \) when \( x = -4 \)

\[
\begin{align*}
  y &= -4^2 + \frac{2}{-4} \\
  &= 16 - \frac{1}{2} \\
  &= 15 \frac{1}{2}
\end{align*}
\]

a. \( y = x^2 + \frac{2}{x} ; x = -4 \)

b. \( y = x^2 + \frac{10}{x} ; x = 15 \)

c. \( y = x^2 + \frac{6}{x} ; x = -6 \)

d. \( y = x^2 + \frac{5}{x} ; x = -10 \)

e. \( y = x^2 + \frac{5}{x} ; x = -10 \)

f. \( y = x^2 + \frac{4}{x} ; x = -16 \)

g. \( y = x^2 + \frac{3}{x} ; x = -9 \)

h. \( y = x^2 + \frac{2}{x} ; x = -8 \)

i. \( y = x^2 + \frac{2}{x} ; x = -2 \)

j. \( y = x^2 + \frac{1}{x} ; x = -2 \)

Problem solving

What is the difference between the value of \( y \) in \( y = x^2 + 2 \), if you first replace \( y \) with 3 and then with -3?

\( y \) is equal to \( x \) squared plus four divided by \( x \). If \( x \) is equal to eight. Substitute and calculate.

\( y \) is equal to \( p \) squared plus two divided by \( p \). If \( p \) is equal to four. Substitute and calculate.

\( y \) is equal to \( b \) squared plus five divided by \( b \). If \( b \) is equal to 10. Substitute and calculate.

\( y \) is equal to \( m \) squared plus three divided by \( m \). If \( m \) is equal to four. Substitute and calculate.

\( y \) is equal to \( n \) squared plus nine divided by \( n \). If \( n \) is equal to three. Substitute and calculate.
Data handling is a cycle. In this you are going to learn about this cycle. The part you are learning about is in green with some notes.

If we need to know something, we have to start by asking questions. What questions do you think we should ask?

Data handling cycle

- Start with a question
- Collect the data
- Represent the data in a graph
- Organise and record data
- Interpret the graph
- Answer questions, predict, pose new questions

Example:

Before collecting any research data you need to know what question or questions you are asking.

A good way of starting is to come up with a hypothesis. A hypothesis is a specific statement or prediction. The research will determine whether it is true or false.

Here are some examples of a hypothesis:
- Everybody in Grade 7 owns a cell phone.
- All Grade 7s understand square roots.
- All Grade 7s like junk food.
1. Where would you look to find data to give you answers to these questions?

<table>
<thead>
<tr>
<th>Question</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. What is the population of the world?</td>
<td>Primary research</td>
</tr>
<tr>
<td>b. Which learner drinks the most water?</td>
<td>Primary research</td>
</tr>
<tr>
<td>c. What is the rate of population growth in South Africa?</td>
<td>Primary research</td>
</tr>
<tr>
<td>d. What is the population density (number of people per km²) in this town?</td>
<td>Primary research</td>
</tr>
<tr>
<td>e. What languages are spoken in this area?</td>
<td>Primary research</td>
</tr>
<tr>
<td>f. What is South Africa’s most popular food?</td>
<td>Primary research</td>
</tr>
<tr>
<td>g. What is the age structure of the country?</td>
<td>Primary research</td>
</tr>
<tr>
<td>h. What is life expectancy (how long can people expect to live) in South Africa?</td>
<td>Primary research</td>
</tr>
<tr>
<td>i. Which country has the youngest population?</td>
<td>Primary research</td>
</tr>
<tr>
<td>j. What are the most popular foods in this school?</td>
<td>Primary research</td>
</tr>
</tbody>
</table>

2. Is it always possible to collect data directly from the original source?
3. In order to collect the data for Question 1, would you do primary or secondary research or both?

4. Let’s say you want to know the favourite colours of people at your school, but you don’t have the time to ask everyone. How will you go about finding the information?

5. How can we make sure that a result is not biased?

If you only ask people who look friendly, you will only know what friendly people think!

If you go to a swimming pool and you ask people, “Can you swim?”, you will get a biased answer... probably 100% will say “Yes.”

6. How would you design a questionnaire?

A common method of collecting primary data is to use a survey questionnaire.

Questionnaires come in many forms and are carried out using a variety of methods.

The four main methods of conducting a survey using a questionnaire are:

- Face to face
- By post
- By phone
- By internet

There are different ways of designing the questionnaire. You can use:

- Yes/No questions
- Tick boxes for multiple choice questions
- Word responses
- Questions that require a sentence to be written.

Problem solving

How much water do learners in the school drink?

a. Write a hypothesis.

b. How will you find the data to prove or disprove the hypothesis? Will this be primary or secondary data?

c. Find any secondary research data on this topic.

d. Who should you ask?

e. What will the data tell you? (What questions will you ask about the data?)

f. Do you think the data can help you to answer the research question?

g. Think of some appropriate questions. Write them down.

h. Design a simple questionnaire that allows for both Yes/No type responses and multiple-choice responses.
In the previous worksheet we looked at asking a question and collecting data. The next step in the data handling process is to organise the collected data.

We can organise the data using:

- **Tallies**

  Tallying is a way of counting data to make it easy to display in a table. A tally mark is used to keep track of counting.

**Frequency tables**

A frequency table has rows and columns. When the set of data values is spread out, it is difficult to set up a frequency table for every data value as there will be too many rows in the table. So we group the data into class intervals (or groups) to help us organise, analyse and interpret the data.

**Stem–and–leaf tables**

Stem–and–leaf tables (plots) are special tables where each data value is split into “leaf” (usually the last digit) and a “stem” (the other digits). The “stem” values are listed down, and the “leaf” values go right (or left) from the stem values. The “stem” is used to group the scores and each “leaf” indicates the individual scores within each group.

**Example:**

**Frequency table.** Favourite colours for twenty students were as follows:

<table>
<thead>
<tr>
<th>Colour</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purple</td>
<td>/////</td>
<td>4</td>
</tr>
<tr>
<td>Blue</td>
<td>/////</td>
<td>8</td>
</tr>
<tr>
<td>Green</td>
<td>///</td>
<td>3</td>
</tr>
<tr>
<td>Red</td>
<td>/////</td>
<td>5</td>
</tr>
</tbody>
</table>

1. These are marks scored by learners writing a test worth 10 marks.

   6 7 5 7 7 8 7 6 9 7 4 10 6 8 9 5 6 4 8

   Present this information in a frequency table.

**Frequency tables for large amounts of data**

**Example:** The best way to summarise the data in a table or graph is to group the possible options together into groups or categories. So, for example, instead of having 100 rows in our table for exam scores out of 100, we may limit it to five rows by grouping the scores together like this: scores between 0–20; 21–40; 41–60; 61–80; 81–100.

Look at this table of exam scores and compile a tally and frequency table with five categories: 0–20, 21–40, 41–60, 61–80, 81–100.

<table>
<thead>
<tr>
<th>Name</th>
<th>Exam score</th>
<th>Name</th>
<th>Exam score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denise</td>
<td>55</td>
<td>Elias</td>
<td>65</td>
</tr>
<tr>
<td>John</td>
<td>45</td>
<td>Simon</td>
<td>30</td>
</tr>
<tr>
<td>Jason</td>
<td>85</td>
<td>Edward</td>
<td>25</td>
</tr>
<tr>
<td>Mandy</td>
<td>60</td>
<td>Susan</td>
<td>47</td>
</tr>
<tr>
<td>Brenda</td>
<td>79</td>
<td>James</td>
<td>64</td>
</tr>
<tr>
<td>Opelo</td>
<td>59</td>
<td>Nhlanhla</td>
<td>77</td>
</tr>
<tr>
<td>Lisa</td>
<td>53</td>
<td>Lauren</td>
<td>49</td>
</tr>
<tr>
<td>Gugu</td>
<td>90</td>
<td>Tefo</td>
<td>60</td>
</tr>
<tr>
<td>Sipho</td>
<td>63</td>
<td>Alicia</td>
<td>46</td>
</tr>
<tr>
<td>Lerato</td>
<td>51</td>
<td>Betty</td>
<td>73</td>
</tr>
</tbody>
</table>

**Solution**

From this table it is easy to see that most learners scored between 41% and 60% for the exam. Two learners failed the exam, because they scored between 0% and 40% and two learners got distinctions, because they scored between 81% and 100%.

2. The number of calls from motorists per day for roadside service was recorded for a month. The results were as follows:

   28 122 217 130 120 86 80 90 120 140
   70 40 145 187 113 90 68 174 194 170
   100 75 104 97 75 123 100 82 109 120
   81

   Set up a frequency table for this set of data values, using grouped data, grouped in five groups with intervals of 40.
Do at home

1. You collected data by interviewing children in your class about their favourite sport. The results are as follows:

<table>
<thead>
<tr>
<th>Name</th>
<th>Favourite sport</th>
<th>Name</th>
<th>Favourite sport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denise</td>
<td>Netball</td>
<td>Elias</td>
<td>Soccer</td>
</tr>
<tr>
<td>John</td>
<td>Basketball</td>
<td>Simon</td>
<td>Rugby</td>
</tr>
<tr>
<td>Jason</td>
<td>Soccer</td>
<td>Edward</td>
<td>Basketball</td>
</tr>
<tr>
<td>Mandla</td>
<td>Cricket</td>
<td>Susan</td>
<td>Soccer</td>
</tr>
<tr>
<td>Brenda</td>
<td>Cricket</td>
<td>James</td>
<td>Basketball</td>
</tr>
<tr>
<td>Opele</td>
<td>Rugby</td>
<td>Nhlanhla</td>
<td>Rugby</td>
</tr>
<tr>
<td>Lisa</td>
<td>Soccer</td>
<td>Lauren</td>
<td>Tennis</td>
</tr>
<tr>
<td>Gugu</td>
<td>Tennis</td>
<td>Tefo</td>
<td>Rugby</td>
</tr>
<tr>
<td>Sipho</td>
<td>Rugby</td>
<td>Alicia</td>
<td>Soccer</td>
</tr>
<tr>
<td>Lerato</td>
<td>Netball</td>
<td>Betty</td>
<td>Netball</td>
</tr>
</tbody>
</table>

2. You recorded the maximum temperatures per day for the past month. The results are as follows:

- 27
- 28
- 29
- 30
- 31
- 32
- 33
- 34
- 35
- 36
- 37
- 38
- 39
- 40
- 41
- 42
- 43
- 44
- 45
- 46
- 47
- 48
- 49
- 50

a. Set up a frequency table for this set of data values, using grouped data, grouped in six groups with intervals of two.

b. Compile a stem-and-leaf table of the recorded data.

### Organise data continued

3. Compile a stem-and-leaf table of the examination data from the example for Question 1 on the previous page.

**Example:** It will look like this:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

How to split and place 25.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

Now it is easy to see that most learners scored in the 60s - (most leaves).

- Two scored 60 (stem 6 and 2 x leaves of 0), one scored 63, one scored 64 and one scored 65.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5679</td>
</tr>
<tr>
<td>5</td>
<td>1359</td>
</tr>
<tr>
<td>6</td>
<td>00345</td>
</tr>
<tr>
<td>7</td>
<td>379</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>
There are three different types of average that we generally use to understand data:

The **mean** is the total of the numbers divided by how many numbers there are.

The **median** is the middle value.

The **mode** is the value that appears the most often.

We also use the **range** of a set of numbers to see what the difference is between the biggest and the smallest numbers.

**Example:**

<table>
<thead>
<tr>
<th>Height of learners in cm</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>135–140</td>
<td>///</td>
<td>3</td>
</tr>
<tr>
<td>141–145</td>
<td>///</td>
<td>4</td>
</tr>
<tr>
<td>146–150</td>
<td>///</td>
<td>6</td>
</tr>
<tr>
<td>151–155</td>
<td>///</td>
<td>4</td>
</tr>
<tr>
<td>156–160</td>
<td>///</td>
<td>2</td>
</tr>
<tr>
<td>161–165</td>
<td>///</td>
<td>2</td>
</tr>
</tbody>
</table>

First, we need to establish the **range** of the data. The range is the difference between the biggest and the smallest number.

Biggest number = 165

Smallest number = 135

Difference = highest number – smallest number

= 165 – 135

= 30

So the range of this set of numbers is 30.

If we want the width of each class interval to be 5, then the number of groups will be: Range ÷ width of each class = 30 ÷ 5 = 6

So we must divide this set of data into six class intervals (or groups).

From the data and the frequency table, we can establish that the height of the learners ranges from 135 cm to 165 cm. We also know that 21 learners took part in the survey, and that most learners fall into the 146 cm to 150 cm group.

From this data, we can calculate the mean, median, and mode.

**Mean**

The **mean** is the total of the numbers divided by how many numbers there are.

This is the most common average that we normally refer to and which we use to calculate our report cards.

If we add up all 21 numbers in our data range, we will get 3125.

3125 ÷ 21 = 148.8

Therefore the mean for this data range is 148.8.

**Median**

The **median** is the middle value.

In our data range, we have 21 records. To work out the median (middle value), we arrange the data from small to big and then count until the middle value.

The median or middle value in our data range will be the 11th number.

Therefore the median for this data range is 148.

**Mode**

The mode is the value that appears the most often.

Let us arrange the data from small to big:

The value that appears the most is 146.

Therefore the mode for this data range is 146.
1. Use the data set below and calculate the range, the mean, the median and the mode:
3, 13, 7, 5, 21, 23, 39, 23, 40, 23, 14, 12, 56, 23, 29
   a. The range
   b. The mean
   c. The median
   d. The mode

2. Sipho wrote seven maths tests and got scores of 68, 71, 71, 84, 53, 62 and 67. What were the median and mode of his scores?

3. What is the mean of these numbers: 18, 12, 10, 10, 25?

4. The mean of three numbers is 8. Two of the numbers are 11 and 7. What is the third number?

5. The temperature in degrees Celsius over four days in July was 21, 21, 19 and 19. What was the mean temperature?

6. What is the mode of these numbers: 75, 78, 75, 71, 78, 25, 75, 29?

7. Five children have heights of 138 cm, 135 cm, 140 cm, 139 cm and 141 cm. What is the range of their heights?

8. What is the median of these numbers: 2, 4; 2, 8; 2, 3; 2, 9; 2, 9?

9. The cost of five cakes is R28, R19, R45, R45, R15. What is the median cost?

10. What is the range of this group of numbers: 75, 39, 75, 71, 79, 55, 75, 59?

11. What is the median of these numbers: 10, 3, 6, 10, 4, 8?

Do it on your own
These are the test results of 20 learners presented in a stem-and-leaf display.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>5, 6, 7, 9</td>
</tr>
<tr>
<td>5</td>
<td>1, 3, 5, 9</td>
</tr>
<tr>
<td>6</td>
<td>0, 0, 3, 4, 5</td>
</tr>
<tr>
<td>7</td>
<td>3, 7, 9</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: with an even amount of numbers the median will be the value that is halfway between the middle pair of numbers arranged from small to big.

1. Use this data to find the:
   a. Range
   b. Mean
   c. Median
   d. Mode

2. Draw a grouped frequency table showing a tally and frequency column.
2. Critically read and interpret data represented in this bar graph.

Answer the following questions:

1. How many learners are there in the class?

Answer questions, predict, pose new questions

Represent the data in a graph
Collect the data
Interpret the graph
Organise and record data

Steps in drawing a bar graph
1. To draw a bar graph you have to start with your frequency table.
2. From the frequency table decide on the range and scale of the frequency data axis (vertical axis) and the grouped data axis (horizontal axis) and label them.
3. Draw the vertical and horizontal axes and label them.
4. Write the title of the graph at the top.
5. Mark the data on the graph for each data group and draw the bar.
6. Add the colour or shading of the bar to the legend (key), if required to show.

<table>
<thead>
<tr>
<th>Method of transport to school</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>1</td>
</tr>
<tr>
<td>Taxi</td>
<td>2</td>
</tr>
<tr>
<td>Bike</td>
<td>4</td>
</tr>
<tr>
<td>Car</td>
<td>3</td>
</tr>
<tr>
<td>Walk</td>
<td>5</td>
</tr>
</tbody>
</table>

Frequency table:

<table>
<thead>
<tr>
<th>Favourite fruit</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apples</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Oranges</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Grapes</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Bananas</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Kiwi</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Strawberries</td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>
b. Which method of transport is the most popular?


c. Which method is the least popular?


d. How many more learners use the bus than the taxi?


e. Why do you think more learners use the bus than the taxi?


f. Do you think most learners live far from or close to the school?


g. What percentage of the learners use public transport?


Now try it by yourself

Use the data collected during a survey on learners' favourite subjects.

a. Compile a frequency table using tallies.
b. Draw a bar graph using your frequency table.
c. Interpret your graph and write at least five conclusions.

<table>
<thead>
<tr>
<th>Name</th>
<th>Favourite subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peter</td>
<td>Maths</td>
</tr>
<tr>
<td>John</td>
<td>Arts</td>
</tr>
<tr>
<td>Mandla</td>
<td>History</td>
</tr>
<tr>
<td>Bongani</td>
<td>Sciences</td>
</tr>
<tr>
<td>Nandi</td>
<td>Sciences</td>
</tr>
<tr>
<td>David</td>
<td>Maths</td>
</tr>
<tr>
<td>Gugu</td>
<td>History</td>
</tr>
<tr>
<td>Susan</td>
<td>Arts</td>
</tr>
<tr>
<td>Sipho</td>
<td>Maths</td>
</tr>
<tr>
<td>Lebo</td>
<td>Maths</td>
</tr>
<tr>
<td>Ann</td>
<td>History</td>
</tr>
<tr>
<td>Ben</td>
<td>Maths</td>
</tr>
<tr>
<td>Zander</td>
<td>Sciences</td>
</tr>
<tr>
<td>Betty</td>
<td>History</td>
</tr>
<tr>
<td>Lauren</td>
<td>Arts</td>
</tr>
<tr>
<td>Alice</td>
<td>Maths</td>
</tr>
<tr>
<td>Veronica</td>
<td>Language</td>
</tr>
<tr>
<td>Jacob</td>
<td>Maths</td>
</tr>
<tr>
<td>Alicia</td>
<td>History</td>
</tr>
<tr>
<td>Thabo</td>
<td>Language</td>
</tr>
</tbody>
</table>
To record data you can use a double bar graph. A double bar graph is similar to a regular bar graph, but it gives two pieces of related information for each item on the vertical axis, instead of just one.

This type of display lets us compare two related groups of data and make generalisations about the data quickly.

Data handling cycle
- Start with a question
- Organise and record data
- Collect the data in a graph
- Represent the data in a graph
- Interpret the graph
- Answer questions, predict, pose new questions

Example:
The following frequency table shows the number of adult visitors and child visitors to a park.
Construct a side-by-side double bar graph for the frequency table.

<table>
<thead>
<tr>
<th>Visits to the park</th>
<th>Adults</th>
<th>Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>April</td>
<td>300</td>
<td>250</td>
</tr>
<tr>
<td>May</td>
<td>500</td>
<td>350</td>
</tr>
<tr>
<td>June</td>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>July</td>
<td>200</td>
<td>50</td>
</tr>
</tbody>
</table>

Remember:
the two sets of data on a double bar graph must be related.

Visitors to the park

<table>
<thead>
<tr>
<th>April</th>
<th>May</th>
<th>June</th>
<th>July</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adult visitors</td>
<td>300</td>
<td>500</td>
<td>250</td>
</tr>
<tr>
<td>Children visitors</td>
<td>250</td>
<td>350</td>
<td>100</td>
</tr>
</tbody>
</table>

Key/legend
- Adult visitors
- Children visitors

1. The results of exam and practical work by a class is shown in the table below.

<table>
<thead>
<tr>
<th>Name</th>
<th>Practical</th>
<th>Exam</th>
<th>Name</th>
<th>Practical</th>
<th>Exam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denise</td>
<td>60</td>
<td>65</td>
<td>Elias</td>
<td>55</td>
<td>45</td>
</tr>
<tr>
<td>John</td>
<td>63</td>
<td>60</td>
<td>Simon</td>
<td>30</td>
<td>75</td>
</tr>
<tr>
<td>Jason</td>
<td>50</td>
<td>50</td>
<td>Edward</td>
<td>65</td>
<td>59</td>
</tr>
<tr>
<td>Mathelelo</td>
<td>80</td>
<td>75</td>
<td>Susan</td>
<td>65</td>
<td>75</td>
</tr>
<tr>
<td>Beatriz</td>
<td>46</td>
<td>64</td>
<td>Philip</td>
<td>72</td>
<td>75</td>
</tr>
<tr>
<td>Opelo</td>
<td>63</td>
<td>53</td>
<td>Ben</td>
<td>46</td>
<td>72</td>
</tr>
<tr>
<td>Lisa</td>
<td>51</td>
<td>59</td>
<td>Lauren</td>
<td>31</td>
<td>41</td>
</tr>
<tr>
<td>Gugu</td>
<td>67</td>
<td>76</td>
<td>Tefo</td>
<td>75</td>
<td>65</td>
</tr>
<tr>
<td>Sipho</td>
<td>81</td>
<td>80</td>
<td>Alicia</td>
<td>63</td>
<td>58</td>
</tr>
<tr>
<td>Lorato</td>
<td>78</td>
<td>81</td>
<td>Masa</td>
<td>51</td>
<td>53</td>
</tr>
</tbody>
</table>

a. Compile a frequency table using tallies.
b. Draw a double bar graph comparing the learners’ practical marks with their exam marks.

c. Interpret your graph and write down five conclusions.
To record data you can use a histogram. A histogram is a particular kind of bar graph that summarises data points falling in various ranges.

The main difference between a normal bar graph and a histogram is that a bar graph shows you the frequency of each element in a set of data, while a histogram shows you the frequencies of a range of data.

In a histogram the bars must touch, because the data elements we are recording are numbers that are grouped, and form a continuous range from left to right.

Examples of an ordinary bar graph and a histogram:

### Table A

<table>
<thead>
<tr>
<th>Favourite colour</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>///</td>
<td>3</td>
</tr>
<tr>
<td>Red</td>
<td>///</td>
<td>4</td>
</tr>
<tr>
<td>Green</td>
<td>/// /</td>
<td>6</td>
</tr>
<tr>
<td>Yellow</td>
<td>///</td>
<td>4</td>
</tr>
<tr>
<td>Pink</td>
<td>///</td>
<td>2</td>
</tr>
<tr>
<td>Purple</td>
<td>///</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table B

<table>
<thead>
<tr>
<th>Height of learners</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>135–140</td>
<td>///</td>
<td>3</td>
</tr>
<tr>
<td>141–145</td>
<td>///</td>
<td>4</td>
</tr>
<tr>
<td>146–150</td>
<td>///</td>
<td>6</td>
</tr>
<tr>
<td>151–155</td>
<td>///</td>
<td>4</td>
</tr>
<tr>
<td>156–160</td>
<td>///</td>
<td>2</td>
</tr>
<tr>
<td>161–165</td>
<td>///</td>
<td>2</td>
</tr>
</tbody>
</table>

What is the difference between the two frequency tables? In Table A, the frequency covers individual items (Blue, Red, Green, Yellow, Pink and Purple). In Table B the frequency covers a range (135 to 165 – divided into smaller groups, i.e. 135–140, 141–145, 146–150, 141–155, 156–160 and 161–165).

Now let us look at how to construct a histogram.

Let us take the following set of numbers: 3, 11, 12, 19, 22, 23, 24, 25, 27, 29, 35, 36, 37, 45, 49 (We can work out that the mean is 26.5, the median is 24.5, and the mode is 12.)

In most data sets almost all the numbers will be unique and a graph showing how many ones, how many twos, etc. would display data in a meaningful way.

With a histogram, however, we group the data into convenient ranges, called bins. In this example we are going to group the data in bins with a width of 10 each. Changing the size of the bin will change the appearance of the graph.

First we draw a frequency table with the data range divided into the different bins.

<table>
<thead>
<tr>
<th>Data range</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>///</td>
<td>1</td>
</tr>
<tr>
<td>11–20</td>
<td>///</td>
<td>3</td>
</tr>
<tr>
<td>21–30</td>
<td>///</td>
<td>6</td>
</tr>
<tr>
<td>31–40</td>
<td>///</td>
<td>4</td>
</tr>
<tr>
<td>41–50</td>
<td>///</td>
<td>2</td>
</tr>
</tbody>
</table>

Then we tally the data, placing it in the correct bin.
Finally we can draw the histogram by placing the bins on the horizontal axes and the frequency on the vertical axes.

In our example above we can see that most of the data falls within the 21-30 bin and that there is very little deviation from the mean of 26.5 and the median of 24.5.

1. Use the following data to draw a histogram:

30, 32, 11, 14, 40, 37, 16, 26, 12, 33, 13, 19, 38, 12, 28, 15, 39, 11, 37, 17, 27, 14, 36

a. What is the mean, the median and the mode?

b. Complete the frequency table. Make the bins 5 in size ranging from 11 to 40.

c. Draw the histogram.

**Problem solving**

You surveyed the number of times your classmates have travelled to another province. The data you gathered is:

21, 0, 0, 7, 0, 1, 2, 12, 2, 3, 3, 4, 4, 6, 9, 10, 25, 18, 11, 20, 3, 0, 0, 1, 5, 6, 7, 15, 18, 21, 25

Compile a frequency table and then draw a histogram using this data set. Make the bins 3 in size.

What can you tell us about the results of your survey by looking at the histogram?
Part of the power of histograms is that they allow us to analyse extremely large sets of data by reducing them to a single graph that can show the main peaks in the data, as well as give a visual representation of the significance of the statistics represented by those peaks.

This graph represents data with a well-defined peak that is close to the median and the mean. While there are “outliers,” they are of relatively low frequency. Thus it can be said that deviations from the mean in this data group are of low frequency.

1. These two histograms were made in an attempt to determine whether William Shakespeare’s plays were actually written by Sir Francis Bacon. A researcher decided to count the lengths of the words in Shakespeare’s and Bacon’s writings. If the plays were written by Bacon the lengths of words used in these writings should be very similar.

a. What percentage of all Shakespeare’s words are four letters long?

b. What percentage of all Bacon’s words are four letters long?

c. What percentage of all Shakespeare’s words are more than five letters long?

d. What percentage of all Bacon’s words are more than five letters long?

e. Based on these histograms, do you think that William Shakespeare was really just a pseudonym for Sir Francis Bacon? Explain.
2. The two histograms show the sleeping habits of the teenagers at two different high schools. Maizeland High School is a small rural school with 100 learners and Urbandale High School is a large city school with 3,500 learners.

a. About what percentage of the students at Wheatland get at least eight hours of sleep per night?

b. About what percentage of the students at Urbandale get at least eight hours of sleep per night?

c. Which high school has more students who sleep between nine and ten hours per night?

d. Which high school has a higher median sleep time?

e. Wheatland’s percentage of students who sleep between eight and nine hours per night is ________ % more than that of Urbandale.

Problem solving
The table below shows the ages of the actresses and actors who won the Oscar for best actress or actor during the first 30 years of the Academy Awards. Use the data from the table to make two histograms (one for winning actresses’ ages and one for winning actors’ ages). Use bin widths of ten years (0–9, 10–19, 20–29 etc.)

<table>
<thead>
<tr>
<th>Year</th>
<th>Age of winning actress</th>
<th>Age of winning actor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1928</td>
<td>22</td>
<td>42</td>
</tr>
<tr>
<td>1929</td>
<td>36</td>
<td>40</td>
</tr>
<tr>
<td>1930</td>
<td>28</td>
<td>62</td>
</tr>
<tr>
<td>1931</td>
<td>62</td>
<td>53</td>
</tr>
<tr>
<td>1932</td>
<td>32</td>
<td>35</td>
</tr>
<tr>
<td>1933</td>
<td>24</td>
<td>34</td>
</tr>
<tr>
<td>1934</td>
<td>29</td>
<td>33</td>
</tr>
<tr>
<td>1935</td>
<td>27</td>
<td>52</td>
</tr>
<tr>
<td>1936</td>
<td>27</td>
<td>41</td>
</tr>
<tr>
<td>1937</td>
<td>28</td>
<td>37</td>
</tr>
<tr>
<td>1938</td>
<td>30</td>
<td>38</td>
</tr>
<tr>
<td>1939</td>
<td>26</td>
<td>34</td>
</tr>
<tr>
<td>1940</td>
<td>29</td>
<td>32</td>
</tr>
<tr>
<td>1941</td>
<td>24</td>
<td>40</td>
</tr>
<tr>
<td>1942</td>
<td>34</td>
<td>43</td>
</tr>
<tr>
<td>1943</td>
<td>24</td>
<td>49</td>
</tr>
<tr>
<td>1944</td>
<td>29</td>
<td>41</td>
</tr>
<tr>
<td>1945</td>
<td>37</td>
<td>40</td>
</tr>
<tr>
<td>1946</td>
<td>30</td>
<td>49</td>
</tr>
<tr>
<td>1947</td>
<td>34</td>
<td>56</td>
</tr>
<tr>
<td>1948</td>
<td>34</td>
<td>41</td>
</tr>
<tr>
<td>1949</td>
<td>33</td>
<td>38</td>
</tr>
<tr>
<td>1950</td>
<td>28</td>
<td>38</td>
</tr>
<tr>
<td>1951</td>
<td>38</td>
<td>52</td>
</tr>
<tr>
<td>1952</td>
<td>45</td>
<td>51</td>
</tr>
<tr>
<td>1953</td>
<td>24</td>
<td>35</td>
</tr>
<tr>
<td>1954</td>
<td>26</td>
<td>30</td>
</tr>
<tr>
<td>1955</td>
<td>47</td>
<td>38</td>
</tr>
<tr>
<td>1956</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>1957</td>
<td>27</td>
<td>43</td>
</tr>
</tbody>
</table>

Write a short paragraph discussing what your two histograms reveal.
1. Answer the following questions.
   a. Will the sectors always be shown as a percentage? ___________
   b. Will it always add up to 100%? ___________
   c. What was the biggest expense in the South African budget? ___________
   d. What was the smallest expense in the South African budget? ___________

Example: Look at this example of South Africa’s National budget of 2008/9.

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>300</td>
</tr>
<tr>
<td>Food</td>
<td>225</td>
</tr>
<tr>
<td>Transport</td>
<td>75</td>
</tr>
</tbody>
</table>

2. Draw a pie chart that shows the different ingredients of a mushroom pizza (as listed here):
   - Meat 75 g
   - Cheese 250 g
   - Crust 500 g
   - Tomato 125 g
   - Mushrooms 50 g

3. Draw a pie chart to display your expenditure for the week:

<table>
<thead>
<tr>
<th>Expense</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>300</td>
</tr>
<tr>
<td>Food</td>
<td>225</td>
</tr>
<tr>
<td>Transport</td>
<td>75</td>
</tr>
</tbody>
</table>

Waste!

Currently every person in South Africa generates about 2 kg of solid waste per day.

<table>
<thead>
<tr>
<th>Waste category</th>
<th>Waste generated per person per day (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic</td>
<td>240</td>
</tr>
<tr>
<td>Glass</td>
<td>120</td>
</tr>
<tr>
<td>Paper</td>
<td>600</td>
</tr>
<tr>
<td>Metal</td>
<td>200</td>
</tr>
<tr>
<td>Organic</td>
<td>600</td>
</tr>
<tr>
<td>Non-recyclables</td>
<td>240</td>
</tr>
</tbody>
</table>

This table shows the different categories of solid waste and the amount in grams generated per day.

Draw a pie chart to display this information.
1. Use the information from this favourite colour survey and write a report summing up the data and drawing conclusions.

<table>
<thead>
<tr>
<th>Name</th>
<th>Favourite colour</th>
<th>Name</th>
<th>Favourite colour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jacob</td>
<td>Orange</td>
<td>Ann</td>
<td>Red</td>
</tr>
<tr>
<td>John</td>
<td>Blue</td>
<td>Simon</td>
<td>Orange</td>
</tr>
<tr>
<td>Betty</td>
<td>Green</td>
<td>Edward</td>
<td>Blue</td>
</tr>
<tr>
<td>Mandla</td>
<td>Orange</td>
<td>Susan</td>
<td>Blue</td>
</tr>
<tr>
<td>Lebo</td>
<td>Blue</td>
<td>Thabo</td>
<td>Red</td>
</tr>
<tr>
<td>Bongani</td>
<td>Blue</td>
<td>Ben</td>
<td>Blue</td>
</tr>
<tr>
<td>Lisa</td>
<td>Red</td>
<td>Grace</td>
<td>Blue</td>
</tr>
<tr>
<td>Gugu</td>
<td>Blue</td>
<td>Nandi</td>
<td>Red</td>
</tr>
<tr>
<td>Sipho</td>
<td>Green</td>
<td>Wendy</td>
<td>Orange</td>
</tr>
<tr>
<td>Larato</td>
<td>Red</td>
<td>Alice</td>
<td>Green</td>
</tr>
</tbody>
</table>

To report on the data you have collected and analysed you need to remember the shape of a research report:

- Aim
- Hypothesis
- Plan
- Analysis of data
- Interpretation of data
- Conclusions
- Appendices
- References

1. **Aim:**

2. **Hypothesis:**

   A specific statement or prediction that you can show to be true or false.
c. Plan (to collect the data):
What data do you need?

Who will you get it from?

How will you collect it?

How will you record it?

How will you make sure the data is reliable?

Why? Give reasons for the choices you made.

d. Analysis
• This is where you do the calculations and draw charts.
• Graphs are good for representing data visually.
• Note mean and median (not appropriate in this study).
• Note the range as a measure of how spread out the group is (not appropriate in this study).

e. Conclusions:
Do your results agree with the hypothesis?

How confident are you about the results?

What went wrong? How did you deal with it?

What would you do differently if you did the research again?

f. Appendices:
It is good practice to include a copy of the questionnaire if there is one. The appendices may also include tables related to sample selection, instructions to interviewers, and so on.

g. References:
If you used any secondary data or research you must acknowledge your sources here.

Now try this!
Use this favourite subject survey and write a report on the findings. Include a frequency table, graphs and conclusions.

<table>
<thead>
<tr>
<th>Name</th>
<th>Favourite subject</th>
<th>Name</th>
<th>Favourite subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peter</td>
<td>Maths</td>
<td>Ann</td>
<td>History</td>
</tr>
<tr>
<td>John</td>
<td>Arts</td>
<td>Ben</td>
<td>Maths</td>
</tr>
<tr>
<td>Mandla</td>
<td>History</td>
<td>Zander</td>
<td>Sciences</td>
</tr>
<tr>
<td>Bongani</td>
<td>Sciences</td>
<td>Betty</td>
<td>History</td>
</tr>
<tr>
<td>Nandi</td>
<td>Sciences</td>
<td>Lauren</td>
<td>Arts</td>
</tr>
<tr>
<td>David</td>
<td>Maths</td>
<td>Alice</td>
<td>Maths</td>
</tr>
<tr>
<td>Gugu</td>
<td>History</td>
<td>Veronica</td>
<td>Language</td>
</tr>
<tr>
<td>Susan</td>
<td>Arts</td>
<td>Jacob</td>
<td>Maths</td>
</tr>
<tr>
<td>Sipho</td>
<td>Maths</td>
<td>Alicia</td>
<td>History</td>
</tr>
<tr>
<td>Lebo</td>
<td>Maths</td>
<td>Thabo</td>
<td>Language</td>
</tr>
</tbody>
</table>
This assignment will go over two worksheets.

**Is the hand span of Grade 7 girls smaller than that of boys in the same grade?**

**Is there any link between a person’s height and their hand span?**

1. **Choose your research team.**

   **Names of your research team:**

   [List of names]

2. **What is the aim of your research?**

3. **What is your hypothesis?**

4. **Questions that might help you to plan:**
   a. What data do you need?
   b. Who will you get it from?
   c. How will you collect it?
   d. How will you record it?
   e. How will you make sure the data is reliable?

Your group will get an opportunity to present your aim, hypothesis and plan to the rest of the class.

Once all the research teams have presented their plans, you will get the opportunity to change your plans based on what you heard from the other teams.

**Our changes are:**

[Blank space for changes]

**Our revised plan is:**

[Blank space for revised plan]

---

**Data handling cycle**

**Data handling** is a process of collecting, organising, representing, analysing and interpreting data. The visual representation of data is of major importance.

**Preparing**

Now your plan is submitted you should start collecting and recording the data you need.
Interpreting your graphs and writing a report

1. Aim
2. Hypothesis
3. Plan
4. Analysis
5. Interpretation
6. Conclusions
7. Appendices
8. References

Start with a question

Is the hand span of Grade 7 girls smaller than that of boys in the same grade?

Is there any link between a person's height and their hand span?

1. Use the data you collected and recorded to:
   a. Organise your data in a frequency table.
   b. Calculate the mean, the median and the mode.
   c. Calculate the data range.
   d. Draw a stem-and-leaf display.
2. Represent your data in a graph. You may use more than one type of graph.
Possible outcomes

1. a. What is chance you have to land on ____? Write it as a fraction.
   - 3
   - 6
   - 2
   - 1
   - 5
   - 4

1. b. What is your chance to land on ____? Write it as a fraction.
   - Blue
   - Red
   - Purple
   - Orange
   - Yellow
   - Green
   - Blue and yellow
   - Green and red
   - Purple, red and blue
   - Orange, red, blue, green and yellow

1. If the possible outcomes are the following, how many faces will your dice have?
   a. 1, 2, 3, 4, 5, 6, 7, 8
   b. Green, blue, yellow and red
   c. The probability is \(\frac{1}{4}\) to land on 3.
   d. The probability is \(\frac{1}{12}\) to land on 6.

Problem solving

I have a circle that is divided into a number of sectors. Each sector has a number. What could the possible outcomes be for the following:
- circle divided into six equal parts
- circle divided into eight equal parts
- circle divided into two equal parts

The possible outcomes are: 1, 2, 3, 4, 5 and 6.

Possible outcomes

Why are these the possible outcomes?

The possible outcomes are:
- 1, 2, 3, 4, 5 and 6.

1. Why are these the possible outcomes?
   - 1, 2, 3, 4, 5 and 6.

Possible outcomes

What are the possible outcomes?

Terms 4 - Week 3
### Definition of probability

**Problem solving**

What is the probability of a person drawing one sweet from a packet of four sweets? Write it in words, fractions, decimals and percentages.

**This is a probability scale:**

<table>
<thead>
<tr>
<th>unlikely</th>
<th>even chance</th>
<th>likely</th>
</tr>
</thead>
<tbody>
<tr>
<td>impossible</td>
<td>even chance</td>
<td>certain</td>
</tr>
</tbody>
</table>

Read the following statements. Where would you place them on the probability scale?

a. The sun will rise tomorrow.

b. I don't have to study much for maths.

c. When I flip a coin it will land on tails.

When I flip a coin the probability is $\frac{1}{2}$, 0.5 or 50% to land on heads or tails. What does this mean?

We can use words, fractions and/or decimals to show the probability of something happening.

A fraction probability line is shown like this.

---

1. Put these words in the correct place on top of the probability line:
certain, impossible, likely, unlikely, even chance.

2. Put these numbers in the correct place on the probability line:
50%, 75%, 25%, 100% and 0%

Remember that the probability is always expressed as a fraction, percentage or decimal between 0 and 1. e.g. $\frac{1}{2}$, 25% or 0.25 are all ways of saying there is one chance in four.

3. What is the probability of landing on each number on the spinner?

<table>
<thead>
<tr>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

4. Show the following on the probability scale.

Example: The probability to land on 4 on a spinner with four equal sections

5. Write the above as decimals and then percentages.

a. 

b. 

c. 

---

Problem solving

What is the probability of a person drawing one sweet from a packet of four sweets? Write it in words, fractions, decimals and percentages.
Sometimes we cannot tell who will win, but we can look at previous results to estimate the probability.

Let us look at this example: the blue and red teams have played 50 matches.
The red team won 30 of the 50 matches.
The blue team won 10 of the 50 matches.
The two teams drew 10 matches.

- What is the probability of the red team winning the next match?
The chance probability is \( \frac{30}{50} = \frac{3}{5} \) or 60%.

- What is the probability of the blue team winning the next match?
The chance probability is \( \frac{10}{50} = \frac{1}{5} \) or 20%.

This is the formula for relative frequency.

Relative frequency = \( \frac{\text{number of successful trials}}{\text{total number of trials}} \)

1. Calculate the relative frequency.

<table>
<thead>
<tr>
<th>Event</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>A piece of buttered toast was dropped</td>
<td>20 times</td>
<td>16 times with buttered side down</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 times with buttered side up</td>
</tr>
</tbody>
</table>

i. What is the relative frequency for the bread to land with its buttered side down?

\[ \frac{16}{20} = \frac{80}{100} \] or 80%

ii. What is the relative frequency for the bread to land with its buttered side up?

\[ \frac{4}{20} = \frac{20}{100} \] or 20%

b. Coin tossed 100 times

<table>
<thead>
<tr>
<th>Event</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landed 60 times on heads</td>
<td>60 times</td>
<td>( \frac{60}{100} ) or 60%</td>
</tr>
<tr>
<td>Landed 20 times on tails</td>
<td>20 times</td>
<td>( \frac{20}{100} ) or 20%</td>
</tr>
</tbody>
</table>

c. A six-sided dice was rolled 100 times

<table>
<thead>
<tr>
<th>Event</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>The 1 occurred 21 times</td>
<td>21 times</td>
<td>( \frac{21}{100} )</td>
</tr>
<tr>
<td>The 2 occurred 18 times</td>
<td>18 times</td>
<td>( \frac{18}{100} )</td>
</tr>
<tr>
<td>The 3 occurred 17 times</td>
<td>17 times</td>
<td>( \frac{17}{100} )</td>
</tr>
<tr>
<td>The 4 occurred 25 times</td>
<td>25 times</td>
<td>( \frac{25}{100} )</td>
</tr>
<tr>
<td>The 5 occurred 10 times</td>
<td>10 times</td>
<td>( \frac{10}{100} )</td>
</tr>
<tr>
<td>The 6 occurred 9 times</td>
<td>9 times</td>
<td>( \frac{9}{100} )</td>
</tr>
</tbody>
</table>

Problem solving

What is the relative frequency when a drawing pin lands point up 23 times out of 100?
Let us look at the examples and compare.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td></td>
</tr>
<tr>
<td>75%</td>
<td></td>
</tr>
<tr>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

What is the probability of a coin landing on heads?

$\frac{1}{2}$ or 50%

The difference between the probability and the relative frequency is $58\% - 50\% = 8\%$

Will this always be the case?

1. **What is the difference between the probability and relative frequency?** Give your answer in percentages.

   **a.** Dropped a piece of buttered toast 50 times Landed with buttered side down 29 times. 

<table>
<thead>
<tr>
<th>Probability:</th>
<th>Relative frequency:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Difference: ____________

   **b.** Tossed a coin 100 times Landed tails up 52 times. 

<table>
<thead>
<tr>
<th>Probability:</th>
<th>Relative frequency:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Difference: ____________

   **c.** Rolled a 10-sided dice 100 times. Landed 12 times on 5.

<table>
<thead>
<tr>
<th>Probability:</th>
<th>Relative frequency:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Difference: ____________

**Problem solving**

Give 5 everyday life examples of probability.
Revision: number, operations and relationships

In this worksheet we are going to revise number, operations and relationships.

Tick yes or no.

<table>
<thead>
<tr>
<th>Number operations and relationship concepts</th>
<th>Worksheet numbers</th>
<th>Do you need support?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole numbers</td>
<td>R1, R2, R3, R4, R5, R8</td>
<td>Yes</td>
</tr>
<tr>
<td>Exponents</td>
<td>14, 15, 16, 17, 18, 19</td>
<td>Yes</td>
</tr>
<tr>
<td>Integers</td>
<td>105, 106, 107, 108, 109, 110, 111, 112, 113</td>
<td>Yes</td>
</tr>
<tr>
<td>Fractions</td>
<td>Common fractions: R7, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>Decimal fractions: R8, 40, 41, 42, 43, 44, 45, 46, 47</td>
<td>Yes</td>
</tr>
<tr>
<td>Multiples and factors</td>
<td>R6, 5, 6</td>
<td>Yes</td>
</tr>
<tr>
<td>Properties of numbers</td>
<td>R9, 1, 2, 3, 4</td>
<td>Yes</td>
</tr>
<tr>
<td>Financial mathematics</td>
<td>9, 10, 11, 12, 13</td>
<td>Yes</td>
</tr>
<tr>
<td>Ratio and rate</td>
<td>7, 8</td>
<td>Yes</td>
</tr>
</tbody>
</table>

This table will give you information on where to go and revise your work.

My summary and notes.

1. Go through all the worksheets per topic above and make your own notes and summary.

Whole numbers

Exponents

Integers

Fractions

Multiples and factors

Properties of number

Financial mathematics

Ratio and rate

What do you understand now?

After doing this worksheet, share with your teacher and/or friends what you understand now that you didn’t understand before.
Revision: number, operations and relationships continued

Tick yes or no.

<table>
<thead>
<tr>
<th>Patterns, functions and algebra</th>
<th>Worksheet numbers</th>
<th>Do you need support?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Functions and relationships</td>
<td>48, 49, 50, 51, 71, 72, 73, 118, 119</td>
<td>Yes</td>
</tr>
<tr>
<td>Numeric and geometric patterns</td>
<td>65, 66, 67, 68, 69, 70, 114, 115, 116, 117</td>
<td>Yes</td>
</tr>
<tr>
<td>Algebraic expressions</td>
<td>74, 75, 76, 77, 78, 79, 120, 122, 123</td>
<td>Yes</td>
</tr>
<tr>
<td>Algebraic equations</td>
<td>77, 78, 79, 123, 124, 125</td>
<td>Yes</td>
</tr>
<tr>
<td>Graphs</td>
<td>80, 81, 82, 83, 84, 85</td>
<td>Yes</td>
</tr>
</tbody>
</table>

My summary and notes.

1. Go through all the worksheets per topic above and make your own notes and summary.

What do you understand now?

After revising this lesson, share with your teacher and/or friends what you understand now that you didn’t understand before.
In this worksheet we are going to revise shape and space (geometry).

This table will give you information on where to go and revise your work.

<table>
<thead>
<tr>
<th>Shape and space (geometry)</th>
<th>Worksheet numbers</th>
<th>Do you need support?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction of geometric figures</td>
<td>R10, 20, 21, 22, 23, 24, 25</td>
<td>Yes  No</td>
</tr>
<tr>
<td>Geometry of 2-D shapes</td>
<td>21, 22, 24, 26, 27, 28</td>
<td></td>
</tr>
<tr>
<td>Transformation geometry</td>
<td>86, 87, 88, 89, 90, 91, 92, 93, 94</td>
<td></td>
</tr>
<tr>
<td>Geometry of 3-D objects</td>
<td>95, 96, 97, 98, 99, 100, 101, 102, 103, 104</td>
<td></td>
</tr>
</tbody>
</table>

My summary and notes.

1. Go through all the worksheets per topic above and make your own notes and summary.

Constructions of geometric figures

Geometry of 3-D objects

2. Add some everyday life examples for each concept.

Transformation geometry

Geometry of 2-D shapes

Space to make some drawings.

What do you understand now?

After finishing this worksheet, share with your teacher and/or friends what you understand now that you didn’t understand before.
Revision: Measurement

Tick yes or no.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Worksheet numbers</th>
<th>Do you need support?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area and perimeter of 2-D shapes</td>
<td>R12, 52, 53, 54, 55</td>
<td></td>
</tr>
<tr>
<td>Surface area and volume of 3-D objects</td>
<td>R14, 56, 57, 58, 59, 60, 61, 62, 63, 64</td>
<td></td>
</tr>
</tbody>
</table>

My summary and notes.

1. Go through all the worksheets per topic above and make your own notes and summary.

Area and perimeter of 2-D shapes

2. Add some real life examples for each concept.

Surface area and volume of 3-D objects

What do you understand now?

After finishing this worksheet, share with your teacher and/or friends what you understand now that you didn’t understand before.
Revising: data handling

Tick yes or no.

<table>
<thead>
<tr>
<th>Data handling</th>
<th>Worksheet numbers</th>
<th>Do you need support?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collect, organize and summarise data</td>
<td>R16, 126, 127, 128</td>
<td></td>
</tr>
<tr>
<td>Represent data</td>
<td>129, 130, 131, 132</td>
<td></td>
</tr>
<tr>
<td>Analyse, interpret and report data</td>
<td>129, 130, 131, 132, 133, 134, 135, 136</td>
<td></td>
</tr>
<tr>
<td>Probability</td>
<td>137, 138, 139, 140</td>
<td></td>
</tr>
</tbody>
</table>

In this worksheet we are going to revise data handling.

This table will give you information on where to go and revise your work.

My summary and notes.

1. Go through all the worksheets per topic above and make your own notes and summary.

Collect, organize and summarise data

Represent data

2. Add some everyday life examples of data handling.

What do you understand now?

After revising this lesson, share with your teacher and/or friends what you understand now that you didn’t understand before.

Tick yes or no.

Data handling

Worksheet numbers

Do you need support?

Yes

No

Collect, organize and summarise data

R16, 126, 127, 128

Represent data

129, 130, 131, 132

Analyse, interpret and report data

129, 130, 131, 132, 133, 134, 135, 136

Probability

137, 138, 139, 140